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IMPERFECT MERITOCRACY AND INEFFICIENCY

by
ANASTASSIOS D. KARAYIANNIS *

Introduction

The economic system of perfect competition guarantees, more or less, the functioning of the system of perfect meritocracy particularly in the selection of the most efficient individual for a specific job. Meritocracy exists when a society is governed by the following three principles: "(1) A principle of job placement that awards jobs to individuals on the basis of merit; (2) A principle specifying the conditions of opportunity under which the job placement principle is applied; and (3) A principle specifying reward schedules (salary, benefits, etc.) for jobs" (Daniels, 1991, p. 154).

Leaving behind the neoclassical world of perfect competition, imperfect meritocracy could emerge through the individualistic behaviour of politicians and/or because of a false institutional structure. Apart from the various negative effects produced in the economy by bureaucracy, some others could emerge (serious in some countries) by the selection of managers and labourers in a public firm or organization through an imperfect system of meritocracy. The main argument of this paper is that, if a system of employee and manager selection which is not based entirely upon meritocracy exists in an economy (or in part of it), then labour effort will be at a low level. Or, to put it differently, in this paper we shall try to show that there is another source of inefficiency that is emerging particularly in small economies with large public sectors. This source of inefficiency is produced by the

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failure to select managers and labourers on the base of their appropriate training, skills and knowledge.

In the first section of this paper, the causes of an emerging system of imperfect meritocracy are examined. Then, the consequences of such a system on the allocation of resources and labour effort will be analyzed. In the last section, the significant costs of such a system and its effects on economic growth and the competitiveness of the economy will be presented.

Politicians' Behaviour, Imperfect Meritocracy and Work Effort

Imperfect meritocracy could emerge in the public sector of an economy when there exists some institutional weakness and particularly when the government has the power (by its own right) to choose the top managers and senior officials in any public company, organization and department. The same effect may occur if the government has the power to dismiss all or some of the officers and managers installed by a previous government and does so on the grounds that it needs personnel loyal to its program.

If the ruling political party in government has in effect a monopoly control over the state machine, it must be expected that this party will try to use it as a power base for its re-election. This could be accomplished through the selection and promotion of "our people" (i.e., the party's people) in the state machine through an imperfect meritocracy system. However, it cannot be concluded under the above suppositions that the prevailing system of labourer and manager selection in the public sector is solely based on non-meritocracy (through it may be a possible case), but that this kind of selection may co-exist with meritocracy. In other words, the following analysis is related also to cases where the number of managers and labourers in the public sector chosen under imperfect meritocracy is not very large. A few persons chosen through imperfect meritocracy in strategic places in the hierarchy of the public sector may be enough to cause a decrease in the efficiency of public firms and organizations.

Let us explain how such a system of imperfect meritocracy in the public sector could appear. In general, it could emerge because of the institutional weaknesses in the functions of the state-machine, the political parties and the individualistic behaviour of the politicians, even in a democratic state.

Politicians, according to public choice theory, behave as individuals who are supposed to choose their career by maximizing utility. Or as Downs (1957, p. 28) put it:

"From the self-interest axiom springs our view of what motivates the political actions of party members. We assume that they act solely in order to attain the income, prestige, and power which come from being in office. Thus politicians in our model never seek office as a means of carrying out particular policies; their only goal is to reap the rewards of holding office per se. They treat policies purely as means to the attainment of their private ends, which they can reach only by being elected".

Under the above assumption, let us suppose that the utility function of a hypothetical politician is as follows: $U = f(Y, Pr, T)$; where (Y) is total income equal to (Ym) money income plus (Ys) psychic income; (Pr) is the authority exercised upon others as a consequence of their political power; and (T) the time that the politician has been in office. The three factors of the function positively determine the rate of utility that is:

$$\theta U/\theta Y > 0; \qquad \theta U/\theta Pr > 0; \qquad \theta U/\theta T > 0$$

The possibility of the politician's re-election (i.e., factor T) is a function of his political friends and supporters. It is assumed that a number of voters vote for their favorite politician because of his abilities and his political beliefs. Another section of voters vote because their economic interests are better served by the specific politician or political party. In other words, some of the politician's supporters are those who directly and/or indirectly have been favoured by the politician. In some democracies, this kind of supporter may outnumber those who vote according to the capabilities and political beliefs of the candidate.

The number of votes received by a specific politician depends on the power which he has in the government, which in turn is translated into the number of services that he offers to his voters. These services may consist of various governmental subsidies, and/or favourable regulations, but primarily they are related to the number of employment positions (Lm) offered in the public sector to his friends and supporters [1]. A politician's supporters receive "good service" from him particularly when the candidate chosen for a position in the public sector is incompetent in comparison with other candidates. Therefore, as the number of incompetent persons the politician employs in the public sector is increased, his possibility of re-election is also increased, defined as $T = f(Lm)$. Thus, if it is supposed

[1] On the causes and effects of public regulation of some industries, see Stigler (1975). In the present paper the issue of regulatory policy is left aside. Also, in our analysis we ignore the consequences of group pressures on the government for their private benefit which in the economic literature has been labeled "a rent-seeking process" (Buchanan, Tollison, Tullock, eds., 1980). See also Becker (1985) for an analysis of political behaviour under the pressure of interest groups.

that "power" is nothing more than the dominant political party in a current government and its first target is its own re-election, then it will be obvious that this "power" will try to guarantee loyal "customers" (i.e. voters) by offering them positions in the public sector. In other words, the possibility of non-meritocracy in the public sector may be attributed to weak institutional and administrative rules established by the self-interest of the politicians.

Under such suppositions, the utility function of the politician may become $U = f(Y, Pr, Lm)$, where the amount of (Lm) depends upon the power and position of the politician in the government, i.e., $Lm = f(Pr)$. Thus, the utility function of the politician can finally be defined as: $U = f(Y, Pr)$. The utility of the politician derived from his employment is positively related with his rate of total income and his position and power in the state-machine, that is,

$$\theta U/\theta Y > 0, \qquad \theta U/\theta Lm > 0$$

In this way may be explained the various endeavors of the politicians to increase their social and economic power and authority by expanding the role of the state [2].

In the above analysis is described a hypothetical politician who attempts to maximize his votes. The behaviour of bureaucrats and their influence on economic policies and measures of the state are left aside, as it is supposed here that a part of the bureaucratic structure is directed not by professional bureaucrats, but by servants of the political party that governs the state [3].

Let us examine now the consequences of imperfect meritocracy in the public sector of the economy on the rate of work effort of managers and labourers. The work effort is related with the degree of moral engagement, the level of training, and the material and non-material incentives of the labourer. In other words, the work effort is related with the will and capacity of individuals to accomplish a given task. The work effort has been defined more analytically by Leibenstein as consisting of four elements $(APQT)$:

> "*A* the choice of activities which compose the effort; *P* the pace at which each activity is carried out per unit of time; *Q* the quality of each activity; *T* the time

[2] This target of politicians explains the observation of STIGLER (1975, p. 61): "... the innumerable regulatory actions are conclusive proof, not of effective regulation, but of the desire to regulate".

[3] For bureaucracy and its economic consequences see the introductory analysis of TULLOCK, MCKENZIE (1985, ch. 11).

pattern and length of activity" (Leibenstein, 1976, p. 98; see also Frantz, 1988, p. 75).

Public and private firms are buying labour time, while in their production process they use labour effort which is "to a considerable degree a discretionary variable" (Leibenstein, 1976, p. 157). Firms and other organizations can not pre-set a bundle of effort because of the high cost of its setting and the inefficiencies produced by an elimination of independent actions of men [4]. Thus, if the motives are inefficient and the labourers have been chosen through imperfect meritocracy, their work effort would have a correspondingly low level.

The low level of work effort could be explained in more detail on the following grounds. It is supposed that all men, according to the natural right of independence and autonomy, wish to be free, and not to be coerced and dependent on others [5]. Someone who has been chosen for a position by means of imperfect meritocracy criteria (mainly in the public sector where such criteria may function more easily) must show an obedience to the mechanism or the individual offering him that position. Let us denote this mechanism or individual the "power". Being dependent may create in the labourer an inferiority complex, a complex which may result in either an obedient or a hostile behaviour toward the "power". If the latter behaviour prevails, then the labourer will refuse to obey the directions of the "power". In the event that the first behavior emerges, the dependence and the obedience of the labourer to the "power" in question will be the result of imperfect meritocracy. Or to put it differently, the incompetent labourer who was appointed through imperfect meritocracy will be prisoner to the directions of the "power".

In the case where managers and other officials in the public sector have been chosen under imperfect meritocracy, then an inefficient allocation of public administrators will emerge. This will result in an inefficient allocation of subordinate officials and other personnel in the public sector. The reasons for such a hierarchical inefficient allocation of human resources are the following: (a) subordinates will not have any strong motive to increase their work effort because their promotion will take place through non objective criteria, that is, not in regard to their knowledge and skill (see Vroom, 1964, pp. 152-3); and (b) those officials and managers chosen under an

[4] For the costs and inefficiencies of the effects of pre-set level of labour effort, see LEIBENSTEIN (1976, pp. 100-3); FRANTZ (1988, p. 76).

[5] Upon these principles Houmanidis developed his theory of the cost of dependence, see HOUMANIDIS (1985, 1994); KARAYIANNIS (1990).

imperfect meritocracy system will be unable to increase the efficiency of the organization and the productivity of their subordinates because they lack the necessary skill and knowledge to do so.

The work effort of the labourer is determined not only by the level of material incentives (i.e., wage rate, work time), but also by hierarchical meritocracy. When the principal is more skillful and able than the subordinate the work effort of the latter is increased, because the subordinate will try to reach the level of performance of his principal in order to take his position. Also, the principal will give promotions under meritocracy for the same reasons that he himself was promoted. In the case where a principal has been chosen for reasons other than skill, quality of performance etc. (namely under a system of imperfect meritocracy), then the criteria which he sets for promotions will be based on the same grounds as were used in his own case. In such a situation, the subordinate has no incentive to increase either his work effort or his level of performance.

Moreover, the transmission of motives and objectives from principals to subordinates in the hierarchical scale of state organizations will not guarantee the maximum efficiency. The reason is simple: the incompetence of persons holding higher level positions in the hierarchy will result in low level work effort and thus the work effort of their subordinates will also be less. In other words, the influential effect from the upper to lower scale in the hierarchy will result in a decrease of work effort. Therefore, when the selection of labourers in the public sector takes place under imperfect meritocracy, their productivity will be lower than it would be otherwise [6].

The consequences of such a system of imperfect meritocracy can easily be shown. Let the utility function of the officer and manager of the public firms and organizations be chosen under imperfect meritocracy: $U = f(w, pr, t, ob)$, where (w) is his material rewards; (pr) all the other non material rewards of his position (i.e., power, large office, etc.); (t) the time of occupation of the position; and (ob) the obligation and obedience to the "power". We have that:

$$\theta U/\theta w > 0; \qquad \theta U/\theta pr > 0; \qquad \theta U/\theta t > 0; \qquad \theta U/\theta ob < 0$$

In other words, when the material and other non-material rewards are increased, the utility of the officer and manager will be increased. Also, his utility directly depends on the length of time of his position.

[6] As LEIBENSTEIN (1976, p. 380) mentioned:

"Certainly improved worker selection could improve productivity at the plant level. To the extent that people are not working at what they are most proficient at, productivity should rise as a consequence of superior selection methods".

On the other hand, when obligations and obedience are decreased, his utility is increased. Thus, the individual selected by the "power" as an officer and manager in the public sector through non-meritocracy, will do the best he can to decrease his rate of obligation to the "power". Therefore, it would not be expected that the official or manager will choose his subordinates under meritocracy; rather the opposite will be the case for the following reasons: 1) he will choose those which the "power" would have chosen, and 2) by choosing subordinates under imperfect meritocracy he will become the "power" for them, and they, in their turn, will have to show him a degree of obedience and obligation.

It is not unreasonable to assume that the efficiency of the new members of a firm or an organization will emulate, after a time, that of the old members, particularly when the new members become familiar with the culture of the firm or organization (see also Blake & Mouton, 1987, pp. 66-7). Thus, in the case where some of the old members have been elected through imperfect meritocracy, they would have established an organizational culture with large inertia and a high cost of leisure. Namely, the labourers will ask for much higher wages in order to decrease their leisure or to increase their work effort. Thus, it is not necessary to suppose that imperfect meritocracy prevails throughout the public sector. Even if it has been used for the selection and/or promotion of only a few persons in strategic places in the hierarchy of an organization, it is sufficient to cause a diminution in the work effort of others.

In addition, the managers and labourers chosen under imperfect meritocracy will not increase their work effort because the cost of the increased effort will lower the utility derived, thus their inertia will be much larger than that of more qualified persons. On the other hand, their cost of moving to other jobs will be high as their competence is low and thus, their efforts to retain their job through obedience to the "power" that offers them employment will be high [7]. Therefore, if in the public sector a significant proportion of managers and labourers have established a habit of low levels of work effort and norms to retain the status quo, the breaking down of such a situation will need the introduction of strong incentives, such as promotion under a system of meritocracy.

Imperfect Meritocracy and X-Inefficiency

The inefficiency of the production process may be produced by the

[7] The inertia of human behaviour, particularly that associated with work effort, is partly influenced by habits and the established work customs in an organization.

following causes: a) the inefficient mechanism of incentives, that is, the low influence of the causal relationship: motive-effort-reward, on the individual's behaviour; and b) the substitution of the non-material incentives from another mechanism, such as an obedience to the rules of "somebody" (physical or political entity) [8].

The postulate of the imperfect meritocracy system developed here causes X-inefficiency which is not clearly an allocative inefficiency but rather closer to Leibenstein's theory of X-inefficiency produced by an organization functioning under a non perfect system of labourer selection [9].

Whether X-inefficiency is more or less important than the inefficiency produced by the non-optimum allocation of resources is a matter of empirical research and lies outside the scope of this paper [10]. Of course, in the case of public firms and organizations which are monopolies, there will be also a welfare loss attributed to allocative inefficiency. That is, the consumers do not get the desired amount of public goods and services.

The X-inefficiency approach allows for non maximizing behaviour and examines the cause and the consequences of such a postulate. One of the causes of X-inefficiency – as will become clear – is produced by the system of imperfect meritocracy assumed to exist in the public sector. Unlike the private sector, where the forces of perfect competition and the self-interest of entrepreneurs result in the selection of the most efficient labourer, in the public sector there is no such guarantee that the most efficient will be employed. One cause, among others, of the lack of competitiveness in the public sector may be attributed to the inefficient system of selection and promotion of public servants which may be established under the specific suppositions of institutional and political weaknesses. In such a case, there is no reason to expect that those institutional and administrative rules will be the most efficient for the promotion of public welfare. In fact the opposite could well be the case, because, as Coe and Wilbert (1985, p. 15) have noted:

"A related requirement for the successful functioning of a democratic system is that

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[8] This cause has been adequately analyzed by LEIBENSTEIN (1976, 1978).

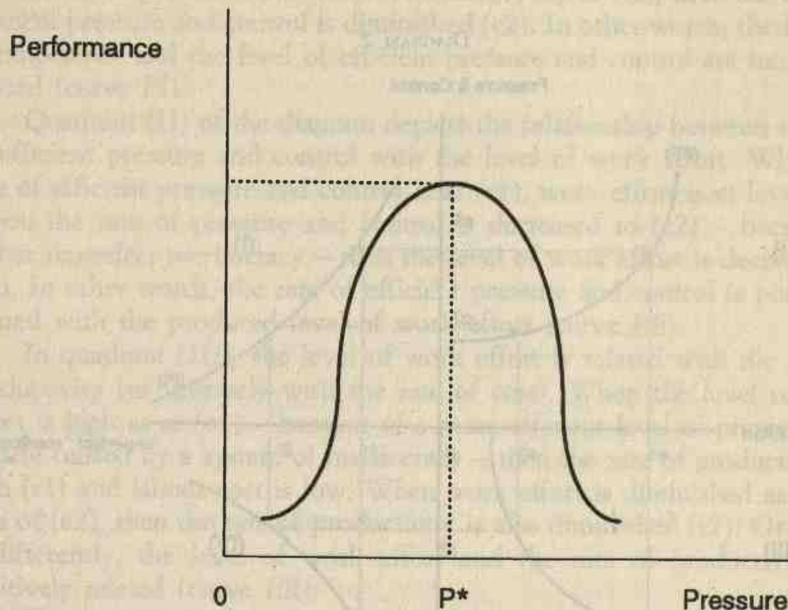
[9] LEIBENSTEIN (1976, p. 45) has attributed X-inefficiency to various causes such as: "contracts for labor are incomplete; not all factors of production are marketed; the production function is not completely specified or known; and interdependence and uncertainty lead competing firms to cooperate tacitly with each other in some respects, and to imitate each other with respect to technique, to some degree".

[10] Total inefficiency could be measured by the difference between the actual output (Q) and the potential output (Q^*) as: $(In) = (Q) - (Q^*)$ (see also FRANTZ, 1988, p. 146).

there exists a well-trained, respected bureaucracy which can insulate itself from short-term public whims".

If this condition is not satisfied, then the economic functioning of the public sector will be inefficient. Let us explain further by using some techniques and propositions of Leibenstein (1976, 1978) regarding the emergence of *X*-inefficiency in the form of a low work effort caused by imperfect meritocracy in the public sector. By relating the level of pressure and performance (see Frantz, 1988, p. 98), we can see from Diagram 1 that the maximum level of performance is attained under an optimum level of pressure P^*.

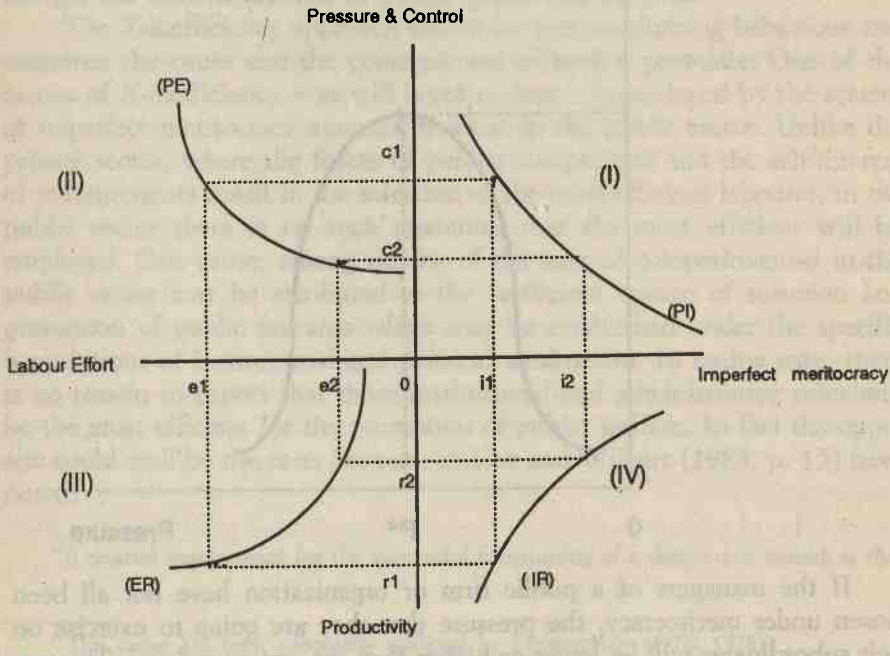
DIAGRAM 1



If the managers of a public firm or organization have not all been chosen under meritocracy, the pressure that they are going to exercise on their subordinates will be lower or higher than the optimum level. The rate of pressure may be lower than the optimum if the principal officers know that their promotion is not related to their achievements and work performance and they are following the rule "live and let live". The rate of pressure will be higher than the optimum if the principal officers are trying to cover their weakness and incompetence because they have a strong psychological need to demonstrate that they are "somebody". Moreover, the optimum rate of pressure is difficult to be achieved if the principals are showing discriminatory behavior toward their subordinates by following a friendly approval toward those of the same political party.

The managers of public firms and organizations which are mostly monopolies have neither the pressure to attain minimum cost levels nor do they face strong risks in regard to their careers if the monopoly shows a loss [11]. If the effort level in a monopolistic firm or organization is low, the marginal cost will be high and thus increased prices will be paid by the consumer. This is the usual practice in state monopolies which have the power to transmit higher costs to higher prices. Also, there is another source of mismanaging funds in a public firm and organization. Public servants are using the money of somebody else and thus they have little incentive to

DIAGRAM 2



[11] FRANTZ (1988, p. 155) mentioned three causes for which the publicly owned firms are less efficient than the privately owned ones:

"First, the government's taxing powers can be used to subsidize the firm if necessary. Second, the owners of these firms cannot allocate the profits of the firm as freely as they could if the firm were privately owned. That is, the motivation to be *X*-efficient in the hopes of receiving profits is usually lessened. Third, publicly owned firms are often monopolists".

DIPARTIMENTO LABORATORIO SCIENZE COGNITIVE ... ECONO

behave according to the rule of decreasing cost and restricting waste of capital.

It is easy to show that the level of imperfect meritocracy is negatively related with the rate of labourers' productivity. In Diagram 2, quadrant (I) depicts the relationship between imperfect meritocracy and pressure and control inside the organization exercised by the principals to subordinates [12]. In the case where some of the principals in strategic places in the hierarchy have been chosen without objective criteria, then their control over their subordinates would not be efficient. Also, they do not have any motive – as we have explained earlier – to increase the work effort of their subordinates. As we can see from Diagram 2, with a low level of imperfect meritocracy (i1) we have a high level of efficient pressure and control (c1). When the level of imperfect meritocracy is increased, say to (i2), then the level of efficient pressure and control is diminished (c2). In other words, the level of incompetence and the level of efficient pressure and control are negatively related (curve *PI*).

Quadrant (II) of the diagram depicts the relationship between the rate of efficient pressure and control with the level of work effort. When the rate of efficient pressure and control is at (c1), work effort is at level (e1). When the rate of pressure and control is decreased to (c2) – because of higher imperfect meritocracy – then the level of work effort is decreased to (e2). In other words, the rate of efficient pressure and control is positively related with the produced level of work effort (curve *PE*).

In quadrant (III), the level of work effort is related with the rate of productivity (or inversely with the rate of cost). When the level of work effort is high as at (e1) – because of a more efficient level of pressure and control caused by a system of meritocracy – then the rate of productivity is high (r1) and labour cost is low. When work effort is diminished as in the case of (e2), then the rate of productivity is also diminished (r2). Or to put it differently, the level of work effort and the rate of productivity are positively related (curve *ER*).

The low level of productivity is linked to a high level of imperfect meritocracy and this is depicted in quadrant (IV). Low level of productivity (r2) is drawn because of a high rate of imperfect meritocracy (i2). Productivity is increased to (r1) when the level of imperfect meritocracy decreases (i1). Thus, the level of imperfect meritocracy is negatively related with the rate of productivity (curve *IR*). This is in accordance with our previous

[12] A similar but not identical diagram is used in LEIBENSTEIN (1978, pp. 166-7).

analysis of the relationship between productivity and a meritocracy system of labourer selection [13].

Therefore, when the selection of managers and other labourers in the firms and organizations of the public sector is made by politicians, the rate of work effort and innovations – as happened in the prior Soviet Union (see Karayiannis, 1993) – will usually be at very low levels.

Some Consequences of Imperfect Meritocracy

There are some significant direct and/or indirect negative effects produced through the existence of imperfect meritocracy. These could result in a decrease of productivity and competitiveness of the whole economy. These negative effects increase as the size of the public sector increases compared with the economy as a whole. More specifically when a meritocracy system does not prevail in the public sector, then resources will be wasted and a welfare loss will appear, particularly in those public firms and organizations which are monopolies. If the difference in the marginal costs of the competitive firm and public firm (monopoly) is attributed not to other causes but only to the system of imperfect meritocracy, a welfare loss for the community emerges because the most efficient employers have not been elected in the public sector. As Pellanda (1993, pp. 661-2) comments:

> "Public services are produced and sold as a State monopoly where consumers' surpluses are transferred to an expansion of production and to a waste of resources to the only benefit of bureaucrats ... it is not profit which matters for public managers, but the size of the public firm from which they derive political power, higher salaries and possibility of distributing social favours".

Moreover, in the case of an imperfect meritocracy system, the waste of resources and *X*-inefficiency would be large. In other words, we shall not have a Pareto optimum as in the case of perfect competition.

In such an economy, since the public sector plays a strategic role in the economy, its inefficiencies will be transmitted in a variety of ways to the private sector. There are various explanations for this fact:

First, the labourers in the private sector, recognizing that their colleagues in

[13] As VROOM (1964, p. 261) concludes:
"... the level of performance of individual workers is related to the extent to which they believe that their chances of receiving a promotion are related to their level of performance on their job and to the valence of the promotion".

the public sector have a low level of labour effort and a peaceful labourers' life, may try to imitate them. They may attempt through their political and other unions to increase their wage in order to equalize their higher work effort with a higher rate of wage or they may decrease their work effort to the same level of the public sector employee. More than that, if the public servants have a strong labour union which is able to pressure the government for a higher wage level, then the workers unions in the private sector may try to achieve the same level of wages. Thus strikes or higher levels of wages will emerge, something which will result in a spiral effect of increasing wages and prices in the economy.

Second, the raw materials and other utilities of the public sector will have a higher price than they would have otherwise under the system of perfect meritocracy. Thus, ceteris paribus, the competitiveness of the private sector in foreign trade will be decreased.

Third, the institutional and other arrangements of the state will be imperfect as they are established through a defective selection of the most qualified servants [14]. Thus, the economic regulations that have been established may not only be imposed on behalf of some narrow economic interests as Stigler has shown (1975), but also they could have been established in an imperfect way.

In addition to the above mentioned negative effects produced by imperfect meritocracy in the public sector, we have another one with catastrophic consequences for the future: the inefficiency of the public educational system if it is based on an imperfect meritocracy system [15].

Conclusions

As has been shown the inertia and irresponsibility in the workforce will increase inefficiency in the economy. This is obvious particularly in the public sector where, for the sake of votes, it is possible for an imperfect

[14] As Pellanda (1993, p. 664) mentions:

"The bureaucrats they employ to realize their policies often lack the necessary economic knowledge as would enable them to operate with competence. They are in fact not naturally selected by the market like the entrepreneurs of private firms but politically appointed or chosen through public competition rules by juridical not economic requirements".

[15] For a case-study of imperfect meritocracy at the University level, see Roig-Alonso (1994).

meritocracy system to prevail. As a result of such inefficiency, not only are resources wasted, but also the productivity of the public sector is decreased and the competitiveness of the private sector in the foreign market is diminished.

The public sector, if it has been organized under a system of imperfect meritocracy, is internally inefficient and thus produces fewer goods and services at a higher cost than it would otherwise do under a perfect meritocracy system. Thus, a reorganization of the public sector is needed through the establishment of objective criteria for the selection of personnel in the workforce. The main solution therefore is the establishment of institutional rules for decreasing the extent of imperfect meritocracy which prevails in the public sector. This could be achieved through the reduction of the power of politicians and the ruling party in government to influence directly and/or indirectly the criteria with which personnel in public firms and organizations are selected and promoted.

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MERITOCRAZIA IMPERFETTA E INEFFICIENZA

Questo articolo esamina la possibilità che si determini in un'economia una meritocrazia imperfetta e mostra che in questo caso il livello dello sforzo del lavoro sarà basso e vi sarà inefficienza.

La meritocrazia imperfetta può verificarsi nel settore pubblico di una economia quando vi è qualche debolezza istituzionale causata da una condotta individualistica dei politici. In particolare, quando il governo ha il potere (per diritto) di scegliere i massimi dirigenti e funzionari di un'impresa, organizzazione e settore pubblici.

Il basso livello dello sforzo dei lavoratori causato dalla meritocrazia imperfetta aumenta l'inefficienza economica. Questo è ovvio particolarmente nel settore pubblico dove per avere voti può risultare prevalente un sistema di meritocrazia imperfetta. Con questa inefficienza non soltanto si sprecano risorse, ma diminuisce anche la produttività del settore pubblico così come la competitività del settore privato sul mercato estero. Si rende così necessaria una riorganizzazione del settore pubblico attraverso la fissazione di criteri obiettivi per la scelta di dipendenti e dirigenti. Questo si può ottenere attraverso la riduzione del potere dei politici e del partito al governo di influenzare direttamente e/o indirettamente il modo in cui vengono scelti e promossi i dirigenti, i dipendenti statali e i lavoratori nelle imprese e organizzazioni pubbliche.

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MERITOCRAZIA [illegible]

Questo modello esamina la possibilità che si determini in un'economia meritocratica [illegible] importante e [illegible] che in questo caso il livello dello sforzo del lavoro sarà basso e vi sarà inefficienza.

La meritocrazia [illegible] può verificarsi nel settore pubblico di una economia quando vi è qualche [illegible] istituzionale [illegible] da una concezione individualistica [illegible]. In particolare quando il [illegible] di [illegible] [illegible] re pubblico.

Il basso livello dello sforzo dei lavoratori causato dalla meritocrazia [illegible] aumenta l'inefficienza economica. [illegible] re pubblico [illegible] per avere voti può rendere prevalente un sistema di meritocrazia imperfetta. [illegible] settore privato sul mercato [illegible] del settore pubblico [illegible] di criteri [illegible] per la scelta di [illegible] promozioni e [illegible]. Questo [illegible] dei politici e dei partiti al governo di influenzare direttamente [illegible] del bilancio in cui [illegible] indipendenti [illegible] volazioni nelle imprese e organizzazioni pubbliche.

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OPTIMALITY CONDITIONS FOR CONTROL SYSTEMS AND ECONOMIC APPLICATIONS

by
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1. *Introduction*

The study of necessary and sufficient conditions in optimal control is a field of intensive research both in a theoretical and in an applied perspective. A satisfactory set of necessary conditions is provided by the Pontryagin Maximum Principle while sufficient conditions are more difficult to be defined. Things are even more difficult if, applying optimal control techniques to economics, one tries to find a global optimum. To achieve this goal the usual crucial assumption is that the Hamiltonian associated with the problem is jointly pointwise convex in the state and control variables, which is the so called Arrow-Mangasarian condition. This approach can be shortly described in the following way: given an existence theorem, such as the Filippov-Cesari theorem, the Arrow-Mangasarian condition guarantees a unique candidate optimal solution obtained from the Pontryagin Maximum Principle which is then the optimal one. Hence the assumption of convexity on the Hamiltonian plays a double role, it is part of an existence theorem and it implies the uniqueness of the optimal solution.

The set of sufficient conditions described above is very demanding both from an economic and from a mathematical point of view. In the economic applications the Arrow-Mangasarian conditions are imposed by means of some economic law such as diminishing marginal utility, diminishing returns, etc. This way to proceed may be arbitrary and may restrict our attention to a formalization which does not catch some interesting economic phenomena. In the growth theory, for example, the Inada conditions on the

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production function and the conditions required on the utility function to have an interior solution rule out many interesting economic phenomena such as increasing returns, externalities, starvation, etc., see e.g. Romer (1986). The above assumptions are quite restrictive also from a mathematical point of view. The Arrow-Mangasarian condition is a sufficient condition for the convexity of the functional but it is not a necessary condition; in Subsection 3.3 we provide an example where the functional is concave and both the Arrow and the Mangasarian conditions are not satisfied.

Studying nonlinear problems the goal of finding a global optimum can be too difficult to achieve and, once we do not have the global convexity assumption, we are not able to define straight on a global optimum. In this case we have to look, first of all, for local maxima and then, among them, for a global maximum. Following this approach, in Stefani and Zezza (1992b, 1993a, 1993b) new sets of second order necessary and sufficient conditions have been developed to characterize weak local optima. These sets of conditions apply to a general problem in optimal control which is characterized by the presence of end-points equality constraints and by pointwise state-control constraints. They foresee fewer restrictive assumptions on the Lagrangean function than the standard ones. Following this approach the pointwise convexity of the Lagrangean f_0, derived in the optimal control literature from the static optimization approach can be relaxed. Moreover, as we will show later, the definition of convexity is strictly related to the state/control constraints, i.e. the convexity of the functional should be checked through the second variation of the functional along the optimal trajectory restricted to an appropriate subspace defined by the linearization of the constraints. A problem, which is not convex on the whole space, can be convex over the horizon $[0, T]$ along the optimal trajectory with respect to the constraints. When applying these necessary and sufficient conditions to a regular linear quadratic problem one is able to characterize global optima.

A last point to be stressed is that our philosophy is different from the one adopted in other attempts to extend the analysis of some economic problems to nonconvexities. For example in Chichilnisky (1977, 1981) and Romer (1986), in an infinite horizon setting, the authors formalize an existence theorem by means of some compactness assumption and then they define, by means of a Maximum Principle, the optimal solution. Here we take the candidate optimal solution derived from the Pontryagin Maximum Principle and then we check whether the second order conditions for optimality are satisfied.

The paper is organized as follows. In Section 2 we state the problem in a general form. We study an abstract optimal control problem with two kinds of constraints: constraints on the end-points of the state variable and pointwise state-control constraints. In Section 3 we state necessary and sufficient conditions for a weak local minimum by means of the Accessory Minimization Problem (Subsection 3.1). In Subsection 3.2 we describe a Jacobi theory for constrained linear quadratic problems which completes the results. In Subsection 3.3 we provide an example where the above concepts are illustrated. In Section 4 some partial results for the infinite horizon are stated. In the last Section 5 two economic examples are analyzed where the necessary/sufficient conditions apply.

2. *Statement of the Problem*

A problem in optimal control theory can be formulated in an abstract form in the following way. Given a compact interval $I = [0, T]$ and the functions

$$f_0: I \times R^N \times R^M \to R$$

$$f: I \times R^N \times R^M \to R^N$$

$$c_i: R^N \times R^N \to R,\ i = 0, \dots, P$$

$$g: I \times R^N \times R^M \to R^H$$

the goal is to

$$\textit{Minimize } J(\xi, u) = c_0(\xi(0), \xi(T)) + \int_0^T f_0(s, \xi(s), u(s))\, ds$$

over all absolute continuous state variables ξ and measurable control functions u, satisfying

$$\dot{\xi}(t) = f(t, \xi(t), u(t)),\ \xi(0) = x \qquad a.e.\ t \in I, \tag{2.1}$$

$$c_i(\xi(0), \xi(T)) = 0,\ i = 1, \dots, P \tag{2.2}$$

$$g(t, \xi(t), u(t)) = 0 \qquad a.e.\ t \in I \tag{2.3}$$

The state space is R^N and the control space is R^M, there are P state end-points equality constraints and H pointwise state-control equality constraints. Since we are interested in local properties, we could assume that the domains of all the maps are not the whole space but just open sets. We prefer the above notation to emphasize the space where we are working. The optimal control problem described above is quite general and many others can be reduced to it, namely all those including inequality constraints.

Inequality constraints on the state end-points can be included in the above characterization. For example the constraint

$$d(\xi(0), \xi(T)) \geq 0$$

can be represented as an equality constraint of the form (2.2) by introducing a new constant state variable, $\dot{q}(t) = 0$, which does not have to satisfy any boundary condition, and defining the equality constraint in the following way

$$d(\xi(0), \xi(T)) - q^2(0) = 0$$

Time dependent state-control inequality constraints can be handled as the equality constraint (2.3) by introducing new unconstrained control variables. For example, if we have the state-control constraint

$$h(t, \xi(t), u(t)) \geq 0$$

by introducing a new unconstrained control variable $r(t)$, it can be written as

$$h(t, \xi(t), u(t)) - r^2(t) = 0$$

Let us now examine the different components of the optimal control problem and their economic origins. The function $c_0(\xi(0), \xi(T))$ defines the cost associated with the state variable initial and final values; this characterization includes, as particular case, the scrap value on $\xi(T)$, a very useful way in economic applications to describe the economic cost associated with the final level of the state variable. There can also be a cost on the initial value of the state variable when we consider an enlarged state space which includes announcement variables, etc.

The functions $c_i(\xi(0), \xi(T)) = 0$, $i = 1, ..., P$, represent in a compact way every end-point constraint. This constraint is a finite dimensional constraint. Usually in economic problems the initial state value is fixed, but more general boundary constraints can be obtained if the state space is enlarged.

Finally the zero-level set of the function $g(t, \xi(t), u(t))$ represents the feasible subset at time t of the state-control space; the constraint is a time dependent state-control constraint and it can be seen as an infinite dimensional constraint. This particular form of the constraint implies that the feasible subset is given by a finite number of qualities. In the economic applications, many times, the values assumed by the control variables are related to the state variables level, i.e. budget constraint, irreversibility condition, etc.

We are now going to describe the assumptions we need. The regularity properties we will impose on the vector field f (see Assumption 2.1) guarantee the existence and the uniqueness of the solution of (2.1) so that we can identify a solution ξ by its initial condition $\xi(0) = x$ and by the control u. In this way the control problem can be seen as an abstract optimization problem on the Banach space

$$E = R^N \times L^\infty (I, R^M)$$

The space E is a Banach space with the norm $||(x, u)||_\infty = ||x|| + ||u||_\infty$.

Definition 2.1: A pair $(x, u) \in E$ is said to be feasible for the above problem if ξ and u satisfy the boundary constraints (2.2) and the state-control constraints (2.3).

Before stating necessary and sufficient conditons we need a little bit of notation. Let Ω be an open subset of E and $\mu : \Omega \to Y$ a C^2-map. $D\mu(e)$ is the Fréchet derivative of the map μ, evaluated at the point e. If $E = E_1 \times E_2 \times \dots \times E_s$, with $E_1, E_2, \dots, E_s$ normed space, we denote $D_i\mu(e)$ the Fréchet derivative with respect to the i-th variable and by $D^2_{ij}\mu(e) = D_i \circ D_j\mu(e) \in \mathcal{L}^2(E_i \times E_j, Y)$, by definition $D^0\mu(e) = \mu(e)$.

To impose the regularity assumptions on the data, the following definition will be used.

Definiton 2.2: Assume that X, Y are finite dimensional vector spaces. We say that the map $G : R \times X \to Y$ is quasi-C^k if it satisfies the following

i) For each $t \in R$, the map $x \to G(t, x)$ is C^k,

ii) The maps $D^i_2 G$, for $i = 0, \dots, k$, are locally essentially bounded and measurable in their variables.

Moreover we will say that the map G is uniformly quasi-C^k if

iii) The map $D^k_2 G$ is continuous in x uniformly with respect to t in any compact interval $J \in R$, *i.e.* for all x_0, $\varepsilon > 0$, there exists $\delta > 0$ such that

$$||x - x_0|| \leq \delta \to ||D^k_2 G(t, x) - D^k_2 G(t, x_0)|| \leq \varepsilon, \ a.e.\ t \in J$$

Remark 2.1: Let us remark that if the function G and its first k derivatives with respect to x are continuous in t then G is uniformly quasi-C^k.

In our case we will require the following

Assumption 2.1.: The maps f, f_0 are quasi C^2, the map g is uniformly quasi-C^2 and the maps c_i, $i = 0, ..., P$, are C^2.

These regularity assumptions are the minimal assumptions to insure that the state flow depends continuously, together with its first and second derivative, on the variables (x, u) of the problem.

A couple $(x_0, \hat{u})$, initial condition and control, is a weak local minimum for the above control problem if it yields a value of the cost which is the minimum with respect to the feasible couples (x, u) which are sufficiently close to it. The difference between weak and strong extrema consists in the topological space considered. Weak extrema refer to the paths which are near to the candidate path in the control space, strong extrema refer to the paths which are near to the candidate path in the state space. The exact definition of weak local minimum is the following

Definition 2.3: A feasible pair $(x_0, \hat{u})$ is said to be a weak local minimum for the above problem if, for some positive ε, $(x_0, \hat{u})$ minimizes J, over all feasible pairs (x, u) satisfying

$$(x, u(t)) \in (x_0, \hat{u}(t)) + \varepsilon B_Q, \ a.e. \ t \in I$$

where B_Q is the unit ball in the space $\mathbf{R}^Q = \mathbf{R}^{N+M}$

From now on we assume that a reference couple $(x_0, \hat{u})$ is given. To this couple it corresponds a state solution $\hat{\xi}$. By " ^ " we denote the evaluation along the reference objects.

3. *Optimality Conditions*

In this section we describe necessary and sufficient conditions for weak local optimality for a nonlinear optimal control problem.

As we pointed out in the Introduction, in the applications of optimal control theory to economics, optimality conditions for weak local extrema have not been considered. Looking for a strong global extremal, the available necessary and/or sufficient theorems are of the Arrow-Mangasarian type (see Kamien and Schwartz, 1971; Seierstad, 1984; Seierstad and Sydsæter, 1977, 1983), and they require a global concavity assumption on the functional.

Our plan is the following, in Subsection 3.1 we state necessary (sufficient) optimality conditions (Theorems 3.1, 3.2) given by the nonnegativity (coerciveness) of a linear quadratic constrained problem, the Accessory Minimization Problem. In Subsection 3.2 we describe a Jacobi theory for this kind of linear quadratic problem which provides, by means of conjugate point (Theorems 3.3, 3.4), necessary and sufficient conditions for its nonnegativity (coerciveness).

3.1. *The Accessory Minimization Problem.* – In this Section we state second order necessary and sufficiency conditions by means of the so called Accessory Minimization Problem which is a linear quadratic constrained problem obtained from the original optimal control problem by means of the first and the second variations.

The study of concavity and second order necessary conditions goes back to the classical works of A.M. Legendre, C.G. Jacobi and K. Weierstrass who first introduced the associated Accessory Minimization Problem and the other second order conditions for the simplest problem in the calculus of variations. Between the two world wars their results have been extended to more general boundary conditions and to the optimal control setting by G.A. Bliss, M. Hestenes et al. (see e.g. Hestenes, 1966).

The presence of state and control constraints makes the problem more complicate. Let us set

$$D(t) \equiv \frac{\partial}{\partial u} g(t, \hat{\xi}(t), \hat{u}(t)) \equiv D_3 \hat{g}(t) \tag{3.1}$$

We suppose that the infinite dimensional constraint is regular at $(x_0, \hat{u})$ by assuming

Assumption 3.1.: The constraint g satisfies the following rank condition at $(x_0, \hat{u})$

$$\det(D(t)\, D^T(t)) \geq K > 0, \; a.e. \; t \in I$$

for some positive $k \in R$.

This assumption allows us to reduce the constraint to a finite dimensional one so that we can give an explicit expression of the modified Hamiltonian associated with the constrained problem.

We set, with the same notations as in (3.1),

$$A(t) \equiv D_2 \hat{f}(t), \; B(t) \equiv D_3 \hat{f}(t), \; B_0(t) \equiv D_3 \hat{f}_0(t), \; C(t) \equiv D_2 \hat{g}(t)$$

and, for sake of simplicity, we denote by ∇ the derivative with respect to the coupled variables $(x, u) \in R^N \times R^M$. From now on all the equations between L^P functions are assumed to hold almost everywhere.

Denoting by $x_1 = \hat{\xi}(T)$ the final value of the reference state trajectory, the linearization of the nonlinear optimal control problem along the reference couple $(x_0, \hat{u})$ is

$$\dot{\xi}_L(t) = A(t)\xi_L(t) + B(t)\,u(t), \quad \xi_L(t_0) = x \tag{3.2}$$

$$Dc_i(x_0, x_1)(x, \xi_L(T)) = 0, \; i = 1, \dots, P \tag{3.3}$$

$$C(t)\,\xi_L(t) + D(t)\,u(t) = 0 \tag{3.4}$$

We will denote the solutions of the above equation by $\xi_L(\cdot, x, u)$. Assumption 3.1 assures the existence of a right inverse of $D(t)$ which can be taken as

$$D^{\#}(t) \equiv D^T(t)\,(D(t)\,D^T(t))^{-1}$$

The optimality conditions can be stated through the Hamiltonian H associated with our problem, modified to take into account the infinite dimensional constraint

$$H : I \times (R^N)^* \times R \times \mathrm{R}^N \times R^M \to R$$

where

$$H(t, \omega, \omega_0, y, w) \equiv \omega\,(f(t, y, w) - B(t)\,D^{\#}(t)\,g(t, y, w)) + \\ + \omega_0\,(f_0(t, y, w) - B_0(t)\,D^{\#}(t)\,g(t, y, w))$$

We can now state the two main theorems on necessary and sufficient optimality conditions. The first theorem holds under two main assumptions: the infinite dimensional constraint is regular (Assumption 3.1) and the multiplier associated with the finite dimensional part (cost and end-point constraints) is unique up to a positive constant. It states that if a reference couple $(x_0, \hat{u})$ is optimal then the usual first order extremality conditions (3.7) hold and the associated Accessory Minimization Problem has zero as a minimum (3.8).

Theorem 3.1: Assume that

i) f, f_0 are quasi-C^2, g is uniformly quasi-C^2 and the c_i, $i = 0, \dots, P$, are C^2,

ii) the rank condition, $\det(D(t)\,D^T(t)) \geq k > 0$, is satisfied.

If $(x_0, \hat{u})$ is a weak local minimum then there exists $\theta = (\theta_0, \ldots, \theta_p) \neq 0$ with $\theta_0 \geq 0$ and a solution λ of the adjoint equation (3.5) and of the transversality conditions (3.6)

$$- \dot{\lambda}(t) = D_4 H(t, \lambda(t), \theta_0, \hat{\xi}(t), \hat{u}(t)) \tag{3.5}$$

$$(-\lambda(t_0), \lambda(t_1)) = \sum_{i=0}^{p} \theta_i Dc_i(x_0, x_1) \tag{3.6}$$

such that the following extremality condition holds

$$D_5 \hat{H}(t) = D_5 H(t, \hat{\lambda}(t), \theta_0, \hat{\xi}(t), \hat{u}(t)) = 0 \tag{3.7}$$

Assume moreover

i) the above multiplier θ is unique up to a positive constant,

then for each (x, u) satisfying the linearized problem (3.2), (3.3), (3.4) we have

$$\sum_{i=0}^{p} \theta_i D^2 c_i(x_0, x_1)(x, \xi_L(T, x, u))^2 + \\ + \int_0^T \nabla^2 \hat{H}(s)(\xi_L(s, x, u), u(s))^2 \, ds \geq 0 \tag{3.8}$$

The second theorem does not require the uniqueness of the multiplier, but the regularity assumption on the functions f and f_0 are stronger. It states that a reference couple $(x_0, \hat{u})$ is optimal if it satisfies the extremality conditions (3.7) and the quadratic form in (3.8) is coercive so that the associated Accessory Minimization Problem has zero as a strict minimum.

Theorem 3.2.: Assume that

i) f, f_0 and g are uniformly quasi-C^2 and the c_i, $i = 0, \ldots, P$, are C^2,

ii) the rank condition, $\det(D(t) D^T(t)) \geq k > 0$, is satisfied,

iii) there exists $\theta = (\theta_0, \ldots, \theta_P) \neq 0$ with $\theta_0 \geq 0$ and a solution $\hat{\lambda}$ of the adjoint equation (3.5) and of the transversality conditions (3.6), for which the first order condition (3.7) is satisfied,

iv) there is $K > 0$ such that for each (x, u) satisfying the linearized problem (3.2), (3.3), (3.4) we have

$$\sum_{i=0}^{P} \theta_i D^2 c_i(x_0, x_1)(x, \xi_L(T, x, u))^2 + \\ + \int_0^T \nabla^2 \hat{H}(s)(\xi_L(s, x, u), u(s))^2 \, ds \geq K \, \| (x, u) \|_2^2$$

then $(x_0, \hat{u})$ is a weak local minimum for the optimal control problem.

The proofs of these theorems are in Stefani and Zezza (1992a, 1992b, 1993b).

Let us remark that the coerciveness condition stated by assumption *iv)* of Theorem 3.2 is established with respect to the L^2 norm of the control, *i.e.* $\|(x, u)\|_2^2 = \|x\|^2 + \|u\|_2^2$; this because a coerciveness property can hold only on a Hilbert space and not on a Banach space.

The quadratic form in (3.8) is a linear quadratic constrained form which defines the Accessory Minimization Problem and it can be explicitly written in the following way

$$I(x, u)^2 = \frac{1}{2}(x^T, \xi_L^T(T))\, \Gamma \begin{pmatrix} x \\ \xi_L(T) \end{pmatrix} +$$

$$+ \frac{1}{2}\int_0^T \{\xi_L^T(s)\, P(s)\, \xi_L(s) + 2u^T(s)\, Q(s)\, \xi_L(s) + u^T(s)\, R(s)\, u(s)\} ds$$

where ξ_L satisfies equation (3.2) and

$$\Gamma = \sum_{i=0}^{p} \theta_i D^2 c_i(x_0, x_1), \qquad \nabla^2 \underline{H}(s) = \begin{pmatrix} P(t) & Q(t) \\ Q^T(t) & R(t) \end{pmatrix}$$

The Accessory Minimization Problem consists in studying the coerciveness or the nonnegativity of the quadratic form I on the closed subspace K of $U = R^N \times L^2(I, \mathrm{R}^M)$ defined by the boundary condition on the state end-points, which can be written as

$$N \begin{pmatrix} x \\ \xi_L(T) \end{pmatrix} = 0$$

where

$$N = Dc\,(x_0, x_1)$$

and the pointwise state control constraint given by (3.4).

In the next subsection we are going to describe a Jacobi theory which can be used to study this type of linear quadratic problems.

3.2. *The Jacobi condition for linear quadratic problems with constraints.* – In the previous section necessary and sufficient conditions for a weak local minimum are stated by means of the properties of a quadratic form defined on a subspace of a Hilbert space. Here we state necessary and sufficient conditions for this quadratic form, defined through a constrained control equation, to be coercive or nonnegative. This, together with the results in the previous section, gives a complete characterization of necessary and/or sufficient conditions for a constrained nonlinear optimal control problem.

The study of linear quadratic problems in the calculus of variations goes back to the work of C.G. Jacobi (1837), some partial extension to control theory are in Mikami (1970) and Dmitruk (1976). In Zezza (1993) an abstract Jacobi theory, obtained by merging some ideas of Hestenes and Poincaré, is developed. By using the results therein a conjugate point approach to linear quadratic problems in presence of different types of equality constraints is obtained in Stefani and Zezza (1993a, 1994b). We consider as Hilbert space the subspace K of U given by the couples (initial state, control) satisfying the constraints. The theory is developed under a surjectivity assumption on the control constraint (Assumption 3.1) and the strengthened Legendre condition (Assumption 3.2). The main results, Theorems 3.3. and 3.4, give a complete characterization of the coerciveness and of the nonnegativeness of the studied quadratic form in terms of semi-conjugate and conjugate points.

When a time dependent matrix acts on a function it becomes an operator between L^p spaces which will be denoted by the same capital letter to have simpler notations. Since the infinite dimensional constraint (3.4) is regular as it is stated by Assumption 3.1, if we define

$$F_1 = \{u \in L^2([0, T], \mathbf{R}^M) : Du = 0\}$$
$$F_2 = \{u \in L^2([0, T], \mathbf{R}^M) : D^\# Du = u\}$$

then F_1 and F_2 give an orthogonal decomposition of $L^2([0, T], \mathbf{R}^M)$ and the corresponding projections are

$$\Pi_1 = Id - D^\# D, \Pi_2 = D^\# D$$

We assume that

Assumption 3.2 The quadratic form I statisfies the strengthened Legendre condition, that is

$$\text{there is } h > 0 \text{ such that } \Pi_1 R \Pi_1 + \Pi_2 \geq h\, Id \tag{3.9}$$

In order to make the statements of the results simpler we reduce our linear quadratic control problem to an equivalent simpler one. Namely, thanks to Assumption 3.1, our problem is equivalent to another one with a functional constraint which does not depend on the state. Consider the quadratic form

$$J(x, u) = (x^T, \zeta^T(T))\,\Gamma \begin{pmatrix} x \\ \zeta(T) \end{pmatrix} + \tag{3.10}$$
$$+ \frac{1}{2}\int_0^T \{\zeta^T(s)\,\bar{P}(s)\,\zeta(s) + 2u^T(s)\,\bar{Q}(s)\,\zeta(s) + u^T(s)\,R(s)\,u(s)\}\,ds$$

on the closed subspace H of U defined by

$$\dot{\zeta} = \bar{A}(t)\,\zeta(t) + B(t)\,u)(t) \tag{3.11}$$

$$N\begin{pmatrix} x \\ \zeta(T) \end{pmatrix} = 0 \tag{3.12}$$

$$Dv = 0 \tag{3.13}$$

where

$$\bar{A} = A - BD^{\#}C \qquad \bar{P} = P - 2C^T(D^{\#})^T Q + C^T(D^{\#})^T RD^{\#}C$$
$$\bar{Q} = Q - RD^{\#}C$$

The equivalence between the two problems can be established.

Lemma 3.1 The quadratic form I is coercive (non-negative) on K if and only if the quadratic form J is coercive (non-negative) on H.

The proof of the equivalence between the two quadratic forms is given by an isomorphism in the control space defined by a feed-back control (see Stefani and Zezza, 1994). Notice that, thanks to the specific feed-back control used, $u \rightarrow u - D^{\#}C\zeta$, all the terms which depend only on the control, u, i.e. B, D, R, remain unchanged.

Since our goal is to state necessary and sufficient conditions in terms of "conjugate points" we need the Jacobi system associated with this problem. The presence of the functional constraint (3.13) modifies the usual system. From Assumption 3.2 it follows that $\Pi_1 R \Pi_1 + \Pi_2$ is invertible. If we set

$$S = (\Pi_1 R \Pi_1 + \Pi_2)^{-1}\,\Pi_1, \tag{3.14}$$

we can now define the Jacobi system which is the Hamiltonian one associated with the Hamiltonian, $\mathcal{H} : \mathbf{R} \times (\mathbf{R}^N)^* \times \mathbf{R}^N \rightarrow \mathbf{R}$, defined by

$$\mathcal{H}(t, \omega, x) = -\frac{1}{2}x^T E(t)\,x + \omega F(t)\,x + \frac{1}{2}\omega G(t)\,\omega^T$$

where $E(t) = \bar{Q}^T(t)\, S(t)\, \bar{Q}(t) - P(t)$, $F(t) = \bar{A}(t) - B(t)\, S(t)\, \bar{Q}(t)$ and $G(t) = -B(t)\, S(t)\, B^T(t)$. The Jacobi system is given by

$$\begin{aligned} \dot{\zeta}(t) &= F(t)\,\zeta(t) + G(t)\,\lambda^T(t) \\ \dot{\lambda}(t) &= \zeta^T(t)\,E(t) - \lambda(t)\,F(t) \end{aligned} \tag{3.15}$$

By means of the solutions of the above system we can characterize the extremals of our problem. Since we are going to consider controls which are zero on $[c, T]$, then on this time interval the corresponding solution of the control system can be expressed through the solution of the matrix equation

$$\dot{\Phi}(t) = \bar{A}(t)\,\Phi(t) \qquad \Phi(c) = Id$$

which will be denoted by $\Phi(t, c)$. On the same interval we can define new boundary conditions and an end-point cost by

$$N_C = N \begin{pmatrix} Id & 0 \\ 0 & \Phi(T, c) \end{pmatrix}$$

$$\Gamma_c = \begin{pmatrix} Id & 0 \\ 0 & \Phi^T(T, c) \end{pmatrix} \Gamma \begin{pmatrix} Id & 0 \\ 0 & \Phi(T, c) \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & 0 \\ 0 & \int_c^T \{\Phi^T(s, c)\,\bar{P}(s)\,\Phi(s, c)\}\, ds \end{pmatrix}$$

In particular we consider the restriction of the quadratic functional J to the subspace corresponding to the zero control, which can be identified with the subspace of $\mathbf{R}^N$.

$$H_0 \equiv \left\{ x \in \mathbf{R}^N : N_0 \begin{pmatrix} x \\ x \end{pmatrix} = 0 \right\}$$

This restriction can be written as the finite dimensional quadratic form

$$J_0 : x \to \frac{1}{2}\,(x^T, x^T)\,\Gamma_0 \begin{pmatrix} x \\ x \end{pmatrix}$$

Definition 3.1: Let $c \in [0, T]$. An absolutely continuous function

$$(\zeta, \lambda) : [0, T] \rightarrow \mathbf{R}^N \times (\mathbf{R}^N)^*$$

is called a c-transversal extremal if it is a solution of the Jacobi system and it satisfies the following transversality conditions

$$N_c \begin{pmatrix} \zeta(0) \\ \zeta(c) \end{pmatrix} = 0$$

$$(-\lambda(0), \lambda(c)) = (\zeta^T(0), \zeta^T(c))\, \Gamma_c + \theta N_c, \text{ for some } \theta \in (\mathbf{R}^P)^*$$

A c-transversal extremal is non-trivial if its state component is non-zero.

Unlike the case of the calculus of variations with one fixed end-point (focal case) here we can have non trivial c-transversal extremals with ζ-subarcs corresponding to the null control, which are described in the following

Definition 3.2: A c-transversal extremal (ζ, λ) is said to be degenerate on $[\alpha, \beta] \subseteq [0, T]$ if $c \in [\alpha, \beta]$ and

$$\Pi_1(t)(\bar{Q}(t)\,\zeta(t) + B^T(t)\,\lambda^T(t)) = 0, \qquad t \in [\alpha, \beta]$$

or, equivalently, for $t \in [\alpha, \beta]$

$$\dot{\zeta}(t) = \bar{A}(t)\,\zeta(t)$$

$$\dot{\lambda}(t) = -\lambda(t)\,\bar{A}(t) - \zeta^T(t)\,\bar{P}(t).$$

We can now introduce the definition of conjugate and semi-conjugate point. We have to make this distinction because of the existence of c-transversal degenerate extremals.

Definition 3.3.: A point $c \in (0, T]$ is called semi-conjugate with zero if there exists a non trivial c-transversal extremal (ζ, λ).

A point $c \in [0, T)$ will be called conjugate with zero if there exists a non trivial c-transversal extremal (ζ, λ) which is not degenerate on $[c, \beta]$.

We are now able to state our main results which correspond to the classical Jacobi necessary and sufficient conditions. Under Assumptions 3.1 and 3.2 the following hold

Theorem 3.3: The quadratic form J is nonnegative if and only if J_0 is nonnegative on H_0 and there is no point $c \in [0, T)$ conjugate with zero.

Theorem 3.4: The quadratic form J is coercive if and only if J_0 is positive on H_0 and there is no point $c \in (0, T]$ semi-conjugate with zero.

These two theorems characterize completely the (strict) concavity of linear quadratic functionals in optimal control. In the next subsection we show that the concavity properties of a functional in optimal control or in the calculus of variations and therefore the existence of a weak local minimum are strictly related to the constraints imposed on the state and on the control. This is quite reasonable since new constraints correspond to a smaller domain of the functional which can be convex-concave on this smaller domain and not on the wider one. Let us remark that the choice of defining (semi)-conjugate points with zero is only one possibility; of course, completely analogous results can be stated for (semi)-conjugate points with T.

3.3. *An example: the harmonic oscillator.* – The following simple example in the calculus of variations, corresponding to the harmonic oscillator, shows how concavity properties of a quadratic functional change with the boundary conditions imposed on the end-points of the state trajectory.

Let us consider the functional

$$J(\zeta) = \frac{1}{2}\zeta^2(T) + \frac{1}{2}\int_0^T \{\dot{\zeta}^2(s) - \zeta^2(s)\}\, ds$$

to be minimized, which is defined on the space of functions having an L^2 integrable derivative, i.e. on the space

$$W^{1,2} = \{\zeta \in AC([0, T], \mathbf{R}^N) : \dot{\zeta} \in L^2([0,T], \mathbf{R}^N)\}$$

The problem can be equivalently written as

$$\text{Minimize } J(x, u) = \frac{1}{2}\zeta^2(T) + \frac{1}{2}\int_0^T \{u^2(s) - \zeta^2(s)\}\, ds$$

$$\dot{\zeta}(t) = u(t), \qquad \zeta(0) = x$$

with $(x, u) \in \mathbf{R}^N \times L^2([0, T], \mathbf{R}^N)$. The associated Hamiltonian is

$$H(\zeta, u, \lambda) = \lambda u - \frac{1}{2}\zeta^2 + \frac{1}{2}u^2$$

It is easy to check that the Mangasarian condition is not satisfied, that is the Hamiltonian is not jointly concave in the state/control space. The necessary conditions of the Minimum Principle yield that the optimal control satisfies $u = -\lambda$ and the minimized Hamiltonian becomes

$$\mathcal{H}(\zeta, \lambda) = -\frac{1}{2}\zeta^2 - \frac{1}{2}\lambda^2$$

which is not concave in the state variable as the Arrow sufficient condition requires. As we stressed above the Arrow-Mangasarian sufficient conditions are sufficient for concavity but they are not necessary. To check the concavity of the functional we have to apply the previously described results.

The functional $J(x, u)$ is quadratic and hence it is concave if and only if it is nonnegative, that is $J(x, u) \geq 0$, for all feasible $(x, u) \in \mathbf{R}^N \times L^2([0, T], \mathbf{R}^M)$. We are now going to consider different boundary conditions and we want to know which is the maximal interval, $[0, T]$, on which the functional is nonnegative.

1. Both end-points are fixed, BC_1, $\zeta(0) = 0$, $\zeta(T) = 0$.

2. Only the initial point is fixed, BC_2, $\zeta(0) = 0$.

3. Periodic boundary conditions, BC_3, $\zeta(0) = \zeta(T)$.

4. No constraint is imposed so that the domain is the whole space, BC_4.

In Table 1 we describe the intervals on which the functional is positive and hence concave for each boundary condition. The Jacobi system associated with the problem is

$$\begin{aligned} \dot{\zeta}(t) &= -\lambda(t) \\ \dot{\lambda}(t) &= \zeta(t) \end{aligned}$$

Table 1

THE CONCAVITY OF J DEPENDING ON THE BOUNDARY CONDITIONS

Boundary Conditions	Max. Interval of Concavity
BC_1	$[0, \Pi]$
BC_2	$[0, 3/4\Pi]$
BC_3	$[0, \sin^{-1}(4/5)]$
BC_4	$[0, \Pi/4]$

which is the harmonic oscillator, the existence of a point semi-conjugate with zero corresponds to the existence of a solution of the above system with the following transversality conditions

1. $BC_1 : \zeta(0) = 0,\ \zeta(c) = 0$

2. $BC_2 : \zeta(0) = 0,\ \lambda(c) = \lambda(1 + c - T)\zeta(c)$

3. $BC_3 : \zeta(0) = \zeta(c),\ \lambda(c) = (1 + c - T)\zeta(c) + \lambda(0)$

4. $BC_4 : \lambda(0) = 0,\ \lambda(c) = (1 + c - T)\zeta(c)$

In the above analysis we concentrated our attention on a particular class of extrema: weak local minima; in economics usually conditions for strong extrema and for global extrema are required. As we stressed above the Arrow and the Mangasarian conditions give us conditions for strong global optima with the global concavity of the functional. If we do not have these conditions we have first of all to find the relative weak extrema, then we can check which path is the optimal in the global and strong sense; this can be done by comparing the value of the cost functional for the different solutions satisfying Theorems 3.1 and 3.2. A theory for strong local minima corresponding to the classical risults of K. Weierstrass is not yet completely developed. We mention Zeidan (1993) where, by means of Riccati techniques, necessary and sufficient conditions for weak and strong local minima are given for a problem in the calculus of variations with separate constraints on the end-points but without other restrictions on the control. A geometric approach is used by Agrachev and Gamkrelidze (1995) where sufficient conditions for strong local optimality are given for a control problem with control taking values in a given set and with one fixed end-point.

4. *Optimality on the Infinite Horizon*

In many economic applications, problems are studied on the infinite horizon. In this case the study of the optimal solution is related to the stability properties of the equilibria of the Hamiltonian system defined by the optimal solution obtained from the Pontryagin Minimum Principle. In the infinite horizon setting the concavity assumption introduced by the Arrow or by the Mangasarian condition is enough, as in the finite horizon problem, to establish sufficient optimality conditions. The same thing does not hold in our case; the necessary and sufficiency conditions presented in the sections above do not apply straight on to the optimization horizon $[0, +\infty)$.

An infinite horizon optimal control problem in the Bolza form can be written as

$$\text{Minimize } J_\infty(x, u) = c_0(x) + \int_0^{+\infty} f_0(s, \xi(s), u(s))\, ds, \tag{4.1}$$

over all absolutely continuous functions ξ and measurable control function u, satisfying

$$\dot{\xi}(t) = f(t, \xi(t), u(t)), \qquad \xi(0) = x, \text{ a.e. } t \in [0, +\infty) \tag{4.2}$$

$$g(t, u(t), \xi(t)) = 0, \text{ a.e. } t \in [0, +\infty) \tag{4.3}$$

Following Carlson et al. (1991) many optimality concepts are defined for infinite horizon optimal control problems. Among them, the *Finitely Optimal* definition seems the most appealing following our approach. A reference couple $(x_0, \hat{u})$ is *Finitely Optimal* if the restriction of the cost functional to a finite interval has $(x_0, \hat{u})$ as the optimal solution among all the trajectories with the same right end-point value. The restriction of the cost to $[0, T]$ will be denoted by $J_T(x, u)$. The concept can be defined both in a local and in a global sense. The following optimality definition can be given now.

Definition 4.1: A couple $(x_0, \hat{u})$ satisfying the Pontryagin Minimum Principle with corresponding state trajectory $\hat{\xi}$ is said to be locally Finitely Optimal for the optimal control problem (4.1), (4.2), (4.3) if, for every $T > 0$, there exists a neighborhood of $(x_0, \hat{u})$ in L^2 such that for every feasible couple (x, u) belonging to it with $\xi(T) = \hat{\xi}(T)$, we have

$$J_T(x_0, \hat{u}) \leq J_T(x, u)$$

To check the optimality of a solution of an infinite horizon optimal control problem one has to establish, first of all, by means of the Pontryagin Minimum Principle the conditions that an optimal solution has to satisfy and then to check, by means of Theorems 3.1, 3.2, that, on any interval $[0, T]$, the candidate optimal trajectory is a weak local minimum. Then, among the solutions describing weak local minima, we can look for the absolute minimum.

5. *Two Examples*

In this section we describe two optimal problems provided by the economic literature where the standard necessary and sufficient conditions are not satisfied and the set of necessary and sufficient conditions presented above apply; since the problems are linear quadratic the described conditions lead to global optima.

The first example comes from the optimal growth literature under nonconvexities; we study the optimal saving problem for a consumer having increasing marginal utility under the capital accumulation irreversibility condition. The second example is a political business cycle model with adaptive expectations.

5.1. *Optimal saving with increasing marginal utility and irreversible investments.* – In Barucci and Zezza (1993b, 1994b) we have analyzed the classical optimal saving problem for a consumer with increasing marginal utility in an economy characterized by irreversible investments. If the consumer has a quadratic utility function then the problem over a finite horizon can be seen as the following optimal control problem

$$\text{Maximize } J_T(c) = \frac{1}{2}\int_0^T e^{-\rho s} c^2(s)\, ds$$

$$\dot{k}(t) = \alpha k(t) - c(t), \qquad a.e.\ t \in [0, T]$$

$$0 \leq c(t) \leq \alpha k(t), \qquad a.e.\ t \in [0, T]$$

Where:

– c : consumption

– k: capital, $k(0) = k_0 > 0$

– ρ : discount factor, $\rho > 0$

– α : instantaneous rate of return, $0 \leq \alpha \leq 1$

The functional $J_T(c)$ is not convex in the state-control variables, therefore the Arrow Mangasarian sufficient conditions are not satisfied and the Filippov-Cesari existence theorem does not apply. Also the existence results obtained by Chichilnisky (1977, 1981) and Romer (1986) do not apply to this case because in the optimal saving problem we do not have the convexity and compactness assumption that hold in the optimal growth case.

In Barucci and Zezza (1994a) we have proved that the candidate optimal solution defined by the Pontryagin Maximum Principle, i.e. Pontryagin Extremal, is given by the following Theorem

Theorem 5.1: The Pontryagin Extremal is, depending on the parameters of the model, the following

$$\hat{c}(t) = \begin{cases} 0 & t \in [0,\ t^*) \\ \alpha\hat{k}(t) & t \in [t^*,\ T] \end{cases}, \qquad \hat{k}(t) = \begin{cases} K_0 e^{\alpha t} & t \in [0,\ t^*) \\ K_0 e^{\alpha t^*} & t \in [t^*,\ T] \end{cases}$$

where

$$t^* = \begin{cases} 0 & \text{if } \rho \geq 2\alpha \text{ or if } \rho < 2\alpha \text{ and } T \leq \frac{1}{\rho}\ln\left(\frac{2\alpha}{2\alpha-\rho}\right) \\ T - \frac{1}{\rho}\ln\left(\frac{2\alpha}{2\alpha-\rho}\right) & \text{if } \rho < 2\alpha \text{ and } T > \frac{1}{\rho}\ln\left(\frac{2\alpha}{2\alpha-\rho}\right) \end{cases}.$$

For the proof of the theorem see Barucci and Zezza (1994a).

Let us examine the two cases, the solution may either have a switch, ($t^* > 0$) or not ($t^* = 0$). The economic meaning of the first solution type is that, if the discount rate is low, the instantaneous rate of return high, $\rho < 2\alpha$, and the optimization horizon long enough, then the optimal policy foresees an initial interval of time when the consumer accumulates all the returns from investments and afterwards an interval of time when the consumer consumes all the surplus.

The economic meaning of the second solution type is that if the consumer discounts strongly the future returns and the instantaneous rate of return on the stock of capital is low enough, $2\alpha \leq \rho$, then the consumer consumes all the surplus at each instant of time. This control can be the candidate optimal solution also for $\rho < 2\alpha$, if the optimization horizon is sufficiently small. This means that, given any instantaneous rate of return on the investments associated with a low enough consumer discount rate, there is an optimization horizon $\bar{T} = \frac{1}{2}\ln\left(\frac{2\alpha}{2\alpha - \rho}\right)$ such that if the optimization horizon is smaller, $T < \bar{T}$, then the optimal policy foresees no accumulation and consumption of all the surplus at any time $t \in [0,\ T]$.

As we remarked above, the Arrow-Mangasarian sufficient conditions are not satisfied and the usual existence theorems do not apply, therefore the Pontryagin Maximum Principle gives us only candidates to be optimal solutions. To prove whether or not the candidate optimal solution is the optimal one, we have to check second order conditions.

To apply to the optimal saving problem the necessary and sufficient conditions defined in the previous section, we first simplify the state differential equation by the following change of variables, $v(t) = c(t)\, e^{-\alpha t}$, $z(t) = k(t) e^{-\alpha t}$, and then we transform the obtained inequality state-control constraints, $0 \leq v(t) \leq \alpha z(t)$, into equality constraints; this can be achieved by defining a new set of control variables, $u^T = (q, v, w)$, used to reduce the inequality constraints into two equality constraints in the following way

$$v(t) \geq 0 \text{ becomes } v(t) - w^2(t) = 0$$

$$v(t) \leq \alpha z(t) \text{ becomes } \alpha z(t) - v(t) - q^2(t) = 0$$

To prove that the candidate optimal solution defined by Theorem 5.1 is the optimal one we have to check the strengthened Legendre condition.

Since in the Hamiltonian associated with the optimal saving problem there is no state quadratic term and no mixed term, the second derivatives with respect to the state and with respect to the control and state are zero. Therefore, considering the new variables, the second order conditions concern the quadratic form

$$\frac{1}{2} \int_0^T < R(s)\, u(s),\; u(s) > ds, \tag{5.1}$$

where

$$R(t) = \begin{pmatrix} -2\hat{w}^2(t) \dfrac{\hat{\theta}(t)\, e^{(2\alpha - \rho)t} + \hat{\lambda}(t)}{4\hat{q}^2(t)\, \hat{w}^2(t) + \alpha \hat{z}(t)} & 0 & 0 \\ 0 & e^{(2\alpha - \rho)t} & 0 \\ 0 & 0 & 2\hat{q}^2(t) \dfrac{\hat{\theta}(t)\, e^{(2\alpha - \rho)t} + \hat{\lambda}(t)}{4\hat{q}^2(t)\, \hat{w}^2(t) + \alpha \hat{z}(t)} \end{pmatrix}$$

and $\hat{\lambda}$ is the costate variable associated with the optimal solution defined by Theorem 5.1.

For a problem without constraints the standard strengthened Legendre condition requires the matrix $R(t)$ to be negative. Since the second eigenvalue of this matrix is positive, the condition is not satisfied. But our problem is an optimization problem with state-control equality constraints and therefore the second order necessary condition has to be modified in an appropriate way as we have shown in the previous Sections.

Following the results presented in the Sections above we have to show that the strengthened Legendre condition, Assumption 3.2, is satisfied. In

Barucci and Zezza (1994a) it is shown that this holds true for the quadratic form (5.1). Moreover the Jacobi system (3.15) associated with the optimal saving problem has been studied in Barucci and Zezza (1994a). No semi-conjugate point exists and therefore the Pontryagin Maximum Principle candidate solution provides a weak local maximum which is also a global optimum as it can be seen from the analysis of the convexified problem, see Barucci and Zezza (1994a).

Infinite Horizon. – As we pointed out above the set of necessary and sufficient conditions can also be applied to the infinite horizon context. The optimal saving problem, defined above, in an infinite horizon framework becomes the following

$$\text{Maximize } J_\infty(c) = \frac{1}{2}\int_0^{+\infty} e^{-\rho s} c^2(s)\, ds$$

with the state space dynamics and constraints described in the previous model.

In Barucci and Zezza (1994b) we have shown that the candidate optimal solutions for a weak local maximum are given by the following Theorem

Theorem 5.2 The Pontryagin Extremals over the Infinite Horizon are

1. $\acute{c}(t) = 0, \quad \text{if } \rho \geq 2\alpha \text{ and } k_0 = 1$

2. $\acute{c}(t) \begin{cases} = \alpha k_0 e^{\alpha t} & t \in [0, \ln(k_0)/\alpha) \\ = 0 & t \in (\ln(k_0)/\alpha, +\infty), \\ \in \{0, \alpha k_0 e^{\alpha t}\} & t = \ln(k_0)/\alpha \end{cases} \quad \text{if } \rho \geq 2\alpha, \text{ and } k_0 > 1.$

3. $\acute{c}(t) = \alpha k_0 e^{\alpha t}, \quad \text{if } \rho \geq 2\alpha$

Among the three solutions it is easy to show, see (Barucci and Zezza, 1994b), that the third one is the absolute extremal.

Theorem 5.3. The Infinite Horizon Optimal Saving Problem has a solution, which is unique, only for $\rho > 2\alpha$. The optimal policy foresees consumption for any $t \geq 0$, i.e. $\acute{c}(t) = \alpha k_0 e^{\alpha t}$.

It is interesting to remark that, for some values of the parameters there is a discontinuity between the finite horizon solution and the infinite horizon one. For $\rho > 2\alpha$, being the discount rate high compared to the interest rate, the solution is consumption and no saving at all at any time t, both for the finite and the infinite horizon. For $\rho < 2\alpha$, in the finite horizon setting, if the time horizon is long enough, there is an initial interval of time when the consumer saves everything, $\left[0, T - \frac{1}{\rho}\ln\left(\frac{2\alpha}{2\alpha - \rho}\right)\right]$, and then the consumer consumes everything; in this case the consumer is aware of the life time so that he weights today's saving with future consumption. When we pass to the infinite horizon setting no solution exists, the consumer is aware of the fact that he lives an infinite life and that if the discount rate is low enough, there is not a solution to the optimal saving problem because the utility is increasing with the initial saving period. In this case there is not a solution because the utility function can be made as large as we want by extending the initial saving period.

5.2. *A political business cycle model.* – The model presented in this subsection, analyzed in Barucci and Zezza (1993a), is a version of the standard political business cycle *à la* Nordhaus with a fully quadratic popularity function and adaptive expectations; the macroeconomic model is described in an IS-LM framework with a Phillips curve.

The government cost function is

$$J(p, y) = \int_0^T e^{\rho s}(\alpha_1 p^2(s) - \alpha_2 y^2(s))\, ds \tag{5.2}$$

and the economic system is representend by

$$\begin{aligned}
y(t) &= \bar{y} + b_1(p(t) - p^*(t)) && \text{Phillips Curve}\\
m(t) - p(t) &= y(t) - b_2 i(t) && LM\\
y(t) &= -b_3 r(t) + b_4 g(t) && IS\\
r(t) &= i(t) - p^*(t) && \text{Real Int. Rate}\\
\dot{p}^*(t) &= \eta(p(t) - p^*(t)) && \text{Adaptive Exp.}
\end{aligned}$$

$$T < \infty,\ \rho \geq 0,\ \eta \geq 0,\ \alpha_1,\ \alpha_2 > 0,\ b_{1,\ldots,4} > 0$$

Where

– y, national income growth rate

– $\bar{y}$, full employment national income growth rate

– p, inflation

– m, money supply growth rate

– r, real interest rate

– i, nominal interest rate

– g, fiscal deficit net of interest payment

– T, general election date

– p^*, expected rate of inflation, $p^*(0) = p^*_0$

– ρ, backward discount rate of voters.

We can take as control variable g, as state variable p^* and, since all the other variables can be expressed through these two, after some easy calculations the functional to be minimized becomes

$$J(g) = \int_0^T e^{\rho s} \left\{ \alpha_1 \left(\frac{g(s) + (b_3 + b_1) p^*(s) - \bar{y}}{b_1} \right)^2 - \alpha_2 (b_3 p^*(s) + g(s))^2 \right\} ds$$

subject to

$$\dot{p}^*(t) = \frac{\eta}{b_1} (b_3 p^*(t) + g(t) - \bar{y})$$

The Hamiltonian is the following

$$H(t, \lambda, p^*, g) \equiv e^{\rho t} \left[\alpha_1 \left(\frac{g + (b_3 + b_1) p^* - \bar{y}}{b_1} \right)^2 + \right.$$

$$\left. - \alpha_2 (b_3 p^* + g)^2 \right] + \lambda \frac{\eta}{b_1} (b_3 p^* + g - \bar{y})$$

Setting $\alpha = \alpha_1 - \alpha_2 b_1^2$ and $\gamma(t) = e^{-\rho t} \lambda(t)$, we have shown in Barucci and Zezza (1993a) that the candidate optimal solution, derived from the Pontryagin Minimum Principle, is described by the solution of the following dynamical system

$$\dot{p}^*(t) = -\eta\alpha a_1 p^*(t) - \frac{1}{2}\alpha\eta^2\gamma(t) + \eta\alpha a_2 b_1\bar{y} \qquad (5.3)$$

$$\dot{\gamma}(t) = (\alpha a_1\eta - \rho)\gamma(t) + 2\alpha a_1 a_2 b_1^2 p^*(t) - 2\alpha a_1 a_2 b_1\bar{y}$$

$$p^*(0) = p_0^*, \qquad \gamma(T) = 0$$

The closed form solution can be easily obtained, see Barucci and Zezza (1993a), as a linear combination of exponentials in the eigenvalues of the linear part of the dynamical system (5.3).

As it is straightforward to check, the Arrow-Mangasarian sufficient conditions are not satisfied and therefore we are not able to say that the solution derived from the Pontryagin Minimum Principle is the optimal one. To have a sufficient condition it is enough to show that the Accessory Minimization Problem has zero as a strict minimum. Denoting by $v(t)$ and by $z(t)$ the variations of the control and of the state variable, the quadratic form defining the Accessory Minimization Problem becomes

$$\Psi(v) = \frac{1}{b_1^2}\int_0^T e^{\rho s}[(\alpha^{-1}b_3^2 + 2a_1 b_1 b_3 + a_1 b_1^2)z^2(s) + $$

$$+ \alpha^{-1}(2b_3 z(s)v(s) + v^2(s)) + 2a_1 b_1 z(s)v(s)]\,ds$$

$$\dot{z}(s) = \frac{\eta}{b_1}(b_1 z(s) + v(s)), \qquad z(0) = 0$$

We have to verify that the strengthened Legendre condition is satisfied, i.e. the coefficient of $v^2(s)$ in the cost functional Ψ is positive, which means

$$\alpha^{-1} = a_1 - a_2 b_1^2 > 0$$

From Theorem 3.4, the quadratic form $\Psi(v)$ is positive definite if and only if the Jacobi condition is satisfied. In this specific case the Jacobi condition requires that there exists no point $c \in [0, T)$ semi-conjugate with T, that is there is not a nontrivial solution of the Jacobi system

$$\dot{\zeta}(t) = \alpha a_1\eta\zeta(t) + 2\alpha a_1 a_2 b_1^2 e^{\rho t} q(t)$$

$$\dot{q}(t) = -\eta\alpha a_1 q(t) - \frac{1}{2}\alpha\eta^2 e^{-\rho t}\zeta(t)$$

$$q(c) = 0, \qquad \zeta(T) = 0$$

In Barucci and Zezza (1993a) we have shown that provided that the optimization horizon is short enough, no semi-conjugate point exists and therefore the candidate optimal solution derived from the Pontryagin Minimum Principle is the optimal one if $\alpha > 0$. Let us notice that for this linear quadratic problem the conditions yield that we found a global minimum.

The optimal solution of the model is not of the type of the one obtained by Nordhaus, i.e. the classical political business cycle. The solution shows that the optimal policy for reelection is not necessarily the classical political business cycle but can be, depending on the values of the parameters, a continuously expansive policy, a political business cycle or a cycle dual to the classical one.

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CONDIZIONI DI OTTIMALITÀ PER SISTEMI DI CONTROLLO E APPLICAZIONI ECONOMICHE

Vengono presentate condizioni di ottimalità necessarie e sufficienti per un problema di controllo ottimo caratterizzato da vincoli sulle variabili di stato e controllo e da vincoli sul punto iniziale e finale delle variabili di stato. La convessità punto punto del funzionale da massimizzare nelle variabili di stato e controllo è indebolita rispetto alle condizioni sufficienti per l'ottimalità utilizzate in economia, condizioni di Arrow/Mangasarian per esempio. Le condizioni presentate sono per estremi locali deboli che, nel caso di problemi lineari quadratici, divengono condizioni per estremi globali forti. L'analisi è sviluppata per il caso di ottimizzazione su un orizzonte finito e poi è estesa al caso di ottimizzazione sull'orizzonte infinito. Sono analizzate due applicazioni economiche: risparmio ottimo nel caso di un consumatore caratterizzato da utilità marginale crescente e un modello di ciclo politico economico.

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CONDIZIONI DI OTTIMALITÀ PER SISTEMI DI CONTROLLO E APPLICAZIONI ECONOMICHE

Riassunto

Vengono presentate condizioni di ottimalità necessarie e sufficienti per un problema di controllo ottimo caratterizzato da vincoli sulle variabili di stato e controllo e da vincoli nel punto iniziale e finale delle variabili di stato. Le condizioni sono poste nei termini di un'approssimazione delle variabili di stato e controllo e includono oltre alle condizioni sufficienti per l'ottimalità [illegible] condizioni di Jacobi; [illegible] per esempio che le condizioni [illegible] sono per estremi locali deboli che, nel caso di problemi lineari-quadratici, diventano condizioni che assicurano globali forti. L'analisi è sviluppata per il caso di ottimizzazione su un orizzonte finito e non è estesa al caso di ottimizzazione sull'orizzonte infinito. Sono analizzate due applicazioni economiche; l'ultima nel caso di un consumatore caratterizzato da utilità marginale crescente e un modello di ciclo politico economico.

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LE DINAMICHE SCHUMPETERIANE E LA CRESCITA

di
Umberto Carneglia *

Introduzione

Il saggio si occupa delle dinamiche di *trend* dei Paesi avanzati utilizzando un modello misto (differenziale e alle differenze) monoprodotto. Il modello si richiama alle teorie di P.M. Romer, G.M. Grossman e in qualche modo anche a quelle di R.M. Solow [1]: esso però si distacca notevolmente dai modelli citati, sulla scia delle intuizioni *schumpeteriane*.

Il modello plurisettoriale di Grossman collega essenzialmente la crescita al ritmo di accumulazione (endogena) delle conoscenze tecniche. Questo, a sua volta, dipende, in condizioni d'equilibrio, da varie grandezze: livello di qualificazione della forza lavoro, distribuzione del lavoro qualificato fra settore della ricerca e altri settori, attitudini della forza lavoro, economie di scala, distribuzione del reddito, tasso di interesse (che in qualche modo indica anche propensione al risparmio delle famiglie). Si veda in particolare il cap. V dell'opera citata. L'accumulazione di capitale (gli investimenti), totalmente distinta dall'accumulazione delle conoscenze tecniche, ha nel modello di Grossman una trascurabile influenza sulla crescita.

Nel modello di Solow l'accumulazione di capitale, collegata alla propensione al risparmio delle famiglie, è determinante per il livello d'equilibrio del prodotto pro-capite, mentre non influenza il ritmo di crescita del sistema, legato al progresso tecnico.

Il modello qui proposto istituisce un legame sistematico fra profitto, innovazione (endogena), crescita e investimento: l'accumulazione del capitale, collegata al profitto e all'innovazione, è ritenuta inscindibile dall'accumu-

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* Roma.

[1] Si vedano sull'argomento: Grossman and Helpman (1991); Romer (1986, 1990); Solow (1956).

lazione delle conoscenze tecniche, che è essa stessa accumulazione di capitale, ovvero investimento. Per questa via il modello monoprodotto evidenzia la tendenza dell'investimento e della stessa produzione a smaterializzarsi nel tempo storico, in termini di valore, nei settori avanzati.

Sulla scorta delle intuizioni schumpteriane inoltre il modello indaga dinamiche e ruoli dal lato produttivo in ombra nei modelli citati e in quelli neoclassici in genere: le dinamiche di innovazione e di diffusione dell'innovazione, il ruolo dell'imprenditore (più precisamente della « funzione impresa », totalmente distinta dalla proprietà) nel processo produttivo, il ruolo dell'autofinanziamento d'impresa.

I limiti del modello. – I limiti e le ipotesi semplificatrici del modello sono numerosi; molti di essi sono presenti anche negli altri modelli richiamati. È molto importante evidenziarli fin dall'inizio per stabilire con quali severe restrizioni debba essere inteso il modello e, per converso, in che modo possa essere utilmente impiegato per far luce su taluni specifici aspetti delle dinamiche di *trend*:

– il modello immagina di poter separare le dinamiche di *trend* da quelle di breve periodo di cui non si occupa; la separazione è in parte arbitraria, in quanto nella realtà i due fenomeni sono strettamente intrecciati e si influenzano reciprocamente;

– nei sistemi ipotizzati sono assenti monopoli privati o pubblici, è assente la stessa Pubblica Amministrazione; più esattamente sono assenti talune variabili tipiche ad essa collegate come la spesa pubblica e la tassazione; la bilancia commerciale e quella dei pagamenti sono in costante pareggio;

– è assente l'inflazione, cioè le dinamiche del livello generale dei prezzi, ma sono presenti le dinamiche dei prezzi relativi reali;

– il sistema è in costante equilibrio di piena occupazione;

– la funzione di produzione è unica per cui il modello è monoprodotto, monosettore;

– le funzioni usate sono continue all'interno di intervalli temporali dati e i fattori hanno una perfetta omogeneità interna e sostituibilità reciproca.

Come si vede, le limitazioni sono numerose, talune inoltre sono molto restrittive; riflettendo su di esse, si potrà evitare di incorrere in un errore non raro che è quello di un utilizzo improprio dei modelli.

1. *Le dinamiche di trend*

Elementi essenziali del modello. – Preliminare all'illustrazione del modello è la definizione di alcune grandezze e di alcune funzioni base.

Il tempo storico è suddiviso in intervalli temporali di uguale ampiezza (per semplicità), contrassegnati dalla variabile discreta t. All'interno di essi si svolge una dinamica tipicamente *schumpeteriana* in due tempi: dinamica di innovazione, da cui deriva la formazione di profitto *schumpeteriano* π, dinamica di diffusione dell'innovazione, con graduale erosione (fino all'annullamento) del profitto π a mano a mano che l'innovazione si diffonde nel sistema. L'innovazione avviene mediante investimenti nella ricerca innovativa da parte dell'imprenditore, la diffusione avviene mediante investimenti di applicazione dell'innovazione realizzati dallo stesso imprenditore o dai suoi imitatori.

L'imprenditore ha un triplice ruolo:

– è promotore degli investimenti innovativi in base a una funzione comportamentale (ex ante) di investimento innovativo;

– è promotore della diffusione dell'innovazione in base a una funzione di investimento di diffusione;

– è gestore dell'impresa in concorrenza quando, essendo totalmente eroso il profitto *schumpeteriano* π, il suo compenso specifico si riduce a un margine costante μ che compensa la sua attività di gestore dell'impresa in concorrenza.

L'imprenditore ha dunque distinti ruoli, cui corrispondono specifici compensi, diversi da quelli che gli possano derivare dall'« eventuale » possesso di beni capitali o di fattori primari. La funzione imprenditoriale che genera il profitto è totalmente distinta sul piano teorico e non di rado su quello empirico dalla proprietà dei beni aziendali. L'aggregazione del profitto d'impresa, dell'interesse sul capitale e della rendita sui fattori primari naturali in un'unica categoria è una delle semplificazioni di cui si serve la teoria economica; essa può essere rimossa se ciò è utile.

La scissione del cosiddetto reddito di capitale in tre distinte categorie:

– π, μ = compenso dell'imprenditore

– r = compenso del finanziatore = costo dei prestiti

– R = compenso del titolare di fattori primari

è indispensabile per l'analisi che viene qui proposta. Essa del resto è in linea con i principi di finanza aziendale, in base ai quali l'utile d'impresa in senso proprio è netto degli oneri finanziari non solo contabili, ma anche figurativi, cioè immaginari, sui mezzi propri d'impresa.

Nel nostro modello la rendita R è il compenso dei fattori primari naturali T, la produttività marginale del capitale ($\partial Y / \partial K$) è il « *margine lordo* » da scindere in tasso di profitto π e tasso d'interesse r. Si distingue così il compenso che i capitali avrebbero ottenuto se ceduti a terzi al tasso r di mercato (dei prestiti), dal *surplus* (profitto π) derivante dalla gestione imprenditoriale.

Il modello monoprodotto utilizza la seguente funzione di produzione:

$$Y_t = A_t^{1-\alpha} K_t^{\alpha} L^{\beta} T^{\gamma} \qquad (\alpha + \beta + \gamma = 1)$$

Come di consueto, Y è un flusso, A, K, L, T sono stock.

A_t rappresenta il livello delle conoscenze raggiunto al tempo t; tale livello è il risultato degli investimenti nella ricerca innovativa effettuati fino al tempo t. A_t ha una dinamica di crescita endogena, come si vedrà fra breve.

K_t è la somma degli investimenti (netti) di capitale, materiale o immateriale diffusivo al tempo storico t.

L e T, che rappresentano la forza lavoro e le risorse primarie, sono privi di pedice $_t$ in quanto, per ipotesi semplificatrice, essi sono in quantità limitata invariante nel tempo e utilizzati totalmente, cioè in costante equilibrio di piena occupazione [2].

Il modello è monoprodotto, per cui dalla funzione di produzione si ottiene il prodotto Y nelle seguenti diverse manifestazioni:

A = *know-how* accumulato attraverso gli investimenti nella ricerca;

K = capitale materiale/immateriale necessario a diffondere l'innovazione; ne fa parte il *training* applicativo aziendale oltre alle spese di progettazione in senso lato (utilizzo applicativo di tecniche note);

C = beni di consumo.

Si intende per investimento, così come indica la teoria, l'impiego di risorse materiali-immateriali per l'ottenimento di beni (o servizi) materiali-immateriali destinati ad accrescere la capacità produttiva. L'investimento può consistere nel contesto del modello in « progetti o prototipi » innovativi (ΔA_t) o in progettazione (esecutiva) e realizzazione di beni strumentali già inventati (ΔK_t). I due tipi di investimento sono complementari: l'innovazione non è produttiva di solito senza una serie di investimenti di diffusione [3].

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[2] Ipotesi semplificatrice classica della piena occupazione.

[3] Nella realtà non tutti i tipi di investimento sono riconducibili a un'innovazione e alla sua diffusione, non tutte le innovazioni necessitano di un costoso processo di investimento; la durata degli investimenti innovativi e di tutti quelli di diffusione connessi è variabile. Per queste e altre ragioni le dinamiche illustrate non sono sempre riconoscibili nella realtà; esse

La forma scelta per la funzione di produzione, e in particolare per $A^{1-\alpha}$, è funzionale alle ipotesi formulate circa la relazione dinamica intercorrente fra A e K, come verrà illustrato in seguito.

L'equilibrio di concorrenza. – Ciascun periodo t [4] è suddiviso in due sottoperiodi: $(t — t + \theta)$ $(t + \theta — t + 1)$; si ipotizza che nei punti estremi di ciascun periodo il sistema sia in equilibrio di concorrenza con piena occupazione di tutti i fattori. Pertanto, nei punti estremi, valgono le relazioni simultanee tipiche dell'equilibrio statico *walrasiano*:

$$\partial Y_t/\partial L = \beta A_t^{1-\alpha} K_t^{\alpha} L^{\beta-1} T^{\gamma} = W_t \quad (1.1)$$

$$\partial Y_t/\partial T = \gamma A_t^{1-\alpha} K_t^{\alpha} L^{\beta} T^{\gamma-1} = R_t \quad (1.2)$$

$$\partial Y_t/\partial K = \alpha A_t^{1-\alpha} K_t^{\alpha-1} L^{\beta} T^{\gamma} = r \quad (1.3)$$

Come vedremo fra breve il capitale innovativo A ha anch'esso una remunerazione, ma questa non è definita mediante relazioni di equilibrio statico *walrasiano*, bensì mediante relazioni dinamiche.

L e T, variabili continue ma limitate, sono per ipotesi in equilibrio di piena occupazione non solo nei punti estremi di ciascun periodo t ma anche in qualunque punto intermedio. Esse assumono perciò costantemente il loro valore limite, invariante per ipotesi [5].

Il valore di r è dato esogenamente, in quanto non oggetto di indagine nel presente lavoro.

Normalizzando ($L = 1$, $T = 1$), le relazioni (1.1), (1.2), (1.3) diventano:

$$\beta A_t^{1-\alpha} K_t^{\alpha} = W_t \quad (1.1')$$

$$\gamma A_t^{1-\alpha} K_t^{\alpha} = R_t \quad (1.2')$$

$$\alpha A_t^{1-\alpha} K_t^{\alpha-1} = r \quad (1.3')$$

Come è noto, dalla (1.3) risulta che qualunque investimento ulteriore ΔK farebbe scendere $\partial Y/\partial K$ al di sotto di r generando un tasso di profitto $\pi = \partial Y/\partial K - r$ negativo, per cui nessun imprenditore effettuerebbe inve-

tuttavia ne costituiscono un aspetto distintamente osservabile nei settori innovativi dell'economia.

[4] t intero maggiore di zero.

[5] Una semplificazione tipica dei modelli marginalisti in genere è considerare L e T fattori omogenei e sostituibili infinitamente e istantaneamente. L'ipotesi semplificatrice viene qui mantenuta, aggiungendo che L e T hanno una dimensione limitata.

stimenti ΔK diffusivi al di là del valore d'equilibrio che K assume in t.

Risulta anche che, se K_t avesse un valore inferiore a quello d'equilibrio ($\pi = \partial Y/\partial K - r > 0$), sarebbe conveniente accrescere K_t mediante investimenti ΔK fino a che K_t non raggiungesse il livello d'equilibrio definito dalla (1.3).

Si ipotizza che se per qualche ragione A_t assume un valore $A_t^* > A_t$, le condizioni d'equilibrio (1.1), (1.2) siano costantemente verificate (cioè i valori di W_t e R_t si riallineino istantaneamente), la condizione d'equilibrio (1.3) non sia invece più verificata (il ridimensionamento di K_t non sia istantaneo) e il sistema si trovi in squilibrio $\partial Y/\partial K > r$, ovvero $\partial Y/\partial K - r = \pi > 0$, a causa del livello di K_t.

Dallo squilibrio deriva la possibilità di investimenti ΔK generati dal reinvestimento di profitti πK_t ($\pi = \partial Y_t^*/\partial K_t - r$) [6].

Le dinamiche dal lato della produzione. – Secondo la lezione *schumpeteriana*, una situazione di equilibrio di concorrenza a tasso di profitto nullo come quella rappresentata dalla (1.3) non dura a lungo. Prima o poi, se esistono nel sistema imprenditori dotati di « *animal spirits* » – per usare l'espressione di Keynes – essi impiegano risorse proprie o di prestito negli investimenti innovativi ΔA_t in base alla previsione (ex ante) di ottenere un ritorno pari a ΔA_t più gli interessi più un margine di profitto aggiuntivo.

Il modello ipotizza che ogni volta che il sistema si trova in una situazione di equilibrio di concorrenza, cioè nel punto terminale di un qualunque periodo t, un imprenditore innovatore metta in moto un nuovo circuito schumpeteriano di innovazione-diffusione.

L'innovatore effettua costosi investimenti innovativi ΔA_t, attingendo ai prestiti del sistema finanziario-bancario, sapendo dall'esperienza che è possibile conseguire un ritorno complessivo $\Delta K_t > \Delta A_t$ attraverso una serie di reimpieghi degli utili resi possibili dall'innovazione. I reimpieghi costituiscono investimenti di attuazione e diffusione dell'innovazione.

Il nucleo del modello è costituito dalle seguenti 2 relazioni che operano in sistema:

(1.4) $$A_{t+1} - A_t = (K_t - K_{t-1})\,\zeta$$
equazione comportamentale d'innovazione

(1.5) $$\dot{\bar{K}}(\tau) = (\alpha A_{t+1}^{1-\alpha} K(\tau)^{\alpha-1} - r)\,K(\tau)$$ equazione di diffusione

α, r, ζ sono parametri « dati » per ipotesi semplificatrice.

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[6] $\partial Y^*/\partial K$ è ottenuta sostituendo nella (1.3) A^* ad A.

La (1.5) è un'equazione differenziale che esprime il reinvestimento dei profitti nel 2° semiperiodo, ripetuto finché questi sono maggiori di zero (gli investimenti, accrescendo il livello di K, fanno tendere a zero il saggio di profitto π).

Risolvendo la (1.5) [7] si ricava una relazione d'equilibrio unica e stabile fra K_t e A_t: $K_t = (\alpha/r)^{\frac{1}{1-\alpha}} A_t$, valida in ogni estremo di t.

La (1.4) è un'equazione alle differenze che esprime il comportamento dell'imprenditore innovatore: questi dimensiona il proprio investimento (ΔA_t) nella ricerca innovativa alla somma dei profitti verificatisi nel precedente periodo (tali profitti, reinvestiti, assommano complessivamente a ΔK_{t-1}) attualizzati mediante un fattore di sconto $\zeta = (1 + r)^{-\lambda} (1 + r_s)^{-\lambda}$. ζ tiene conto sia degli oneri finanziari al tasso di mercato r intercorrenti fra investimento ΔA_t e ritorno ΔK_t, sia dell'incertezza $(1 + r_s)^{-\lambda}$ circa il riverificarsi di un ritorno ΔK pari a quello precedente.

Una volta nota la relazione $K_t = (\alpha/r)^{\frac{1}{1-\alpha}} A_t$ (la (1.4) può essere così riscritta (per sostituzione):

$$\Delta A_t = (\alpha/r)^{\frac{1}{1-\alpha}} \zeta \, \Delta A_{t-1} \qquad (\alpha,\ r,\ \zeta \text{ sono dati}) \qquad (1.4')$$

L'equazione del 2° ordine (1.4′) ammette come soluzione la funzione:

$A_t = A_o [(\alpha/r)^{\frac{1}{1-\alpha}} \zeta]^t$ (A_o è una condizione iniziale che può assumere qualunque valore).

Se poniamo per comodità $(\alpha/r)^{\frac{1}{1-\alpha}} \zeta = \varepsilon^{\delta}$, $(\delta = \log [(\alpha/r)^{\frac{1}{1-\alpha}} \zeta])$, A_t può essere così riscritta in modo più compatto: $A_t = A_o \varepsilon^{\delta t}$.

Dalla soluzione $A_t = A_o \varepsilon^{\delta t}$ risulta:

$$A_t/A_{t-1} = \Delta A_t/\Delta A_{t-1} = \varepsilon^{\delta} = (\alpha/r)^{\frac{1}{1-\alpha}} \xi \,{}^{8}$$

[7] Il punto d'equilibrio unico e stabile in grande è ottenibile semplicemente ponendo $\dot{K}(\tau) = 0$ e verificando la stabilità con un diagramma di fase. L'equazione, di tipo bernoulliano, è anche integrabile; risolvendola è possibile trovare $K^e(\tau)$, come limite di $K(\tau)$ soluzione, per $\tau \rightarrow \infty$.

[8] $(\alpha/r)^{\frac{1}{1-\alpha}} \cdot \zeta$ è l'indice di crescita della funzione A_t; $i = (\alpha/r)^{\frac{1}{1-\alpha}} \zeta - 1 = \varepsilon^{\delta} - 1$ è il tasso effettivo annuo di crescita di A_t.

2. *Precisazioni interpretative*

Tipi d'innovazione - Prezzi. – Il modello esamina le innovazioni che accrescono la produttività, nel duplice senso di elevazione della qualità del prodotto [9] o di riduzione della quantità di fattori per unità di prodotto. Non vengono prese in considerazione innovazioni che creino nuovi prodotti senza aumento della produttività [10]. Grossman, pur dando ampio spazio a tale tipo di innovazione, collega fondamentalmente la crescita all'aumento della qualità del prodotto.

Nel modello monoprodotto proposto, il prezzo di prodotti di qualità costante è costante e, fatto uguale a 1, costituisce l'unità di misura dei valori di tutto il sistema [11].

Alla crescita di Y (quantitativa o qualitativa) corrisponde nel modello l'invarianza del prezzo di Y, che è riferito a una unità di prodotto di qualità costante, e l'invarianza della quantità di fattori primari (L, T) impiegati. Le retribuzioni unitarie d'equilibrio dei fattori primari aumentano in quanto aumenta il valore reale del prodotto per unità di fattore.

Il modello ipotizza che i fattori produttivi primari, L e T, siano in quantità limitata e che i fattori produttivi prodotti, K e A, siano illimitatamente espandibili; il tasso d'interesse reale r è per semplicità costante.

Il fattore di attualizzazione $\zeta = (1 + r)^{-\lambda} (1 + r_s)^{-\lambda}$. – Dall'esperienza precedente l'imprenditore ha ricavato le seguenti informazioni:

1) esiste uno sfasamento temporale tra investimento innovativo e ritorno di esso,

2) non è certo che l'investimento abbia un ritorno; se lo ha, non ne è certa la durata né l'ammontare: non sempre l'innovazione ha successo, se lo ha, altri imprenditori imitatori possono inserirsi nel processo di diffusione; altri imprenditori inoltre possono superare la sua innovazione rendendola obsoleta prima della sua completa diffusione.

Per queste e altre ragioni l'imprenditore innovatore dimensiona il proprio investimento innovativo ΔA_t al ritorno ΔK_{t-1}, verificatosi nel circui-

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[9] Secondo la definizione di Grossman, la crescita della qualità può anche essere interpretata come crescita della quantità di servizi offerti all'utilizzatore (GROSSMAN and HELPMAN, 1991, cap. IV, op. cit.).

[10] Queste innovazioni, a parità di fattori, generano un allargamento della gamma produttiva, con maggiore soddisfazione del consumatore; quindi generano un aumento di « valore » del prodotto, che può ben essere interpretato come un aumento del medesimo.

[11] Il modello non prende in considerazione i fenomeni inflattivi (variazioni del livello generale dei prezzi) ma solo le variazioni reali dei prezzi.

to $t_{-1} - t$ precedente, ridotto mediante un fattore $\zeta = (1 + r)^{-\lambda} (1 + r_s)^{-\lambda}$ che è così composto: $(1 + r_s)^{-\lambda}$ è un fattore di sconto probabilistico che esprime la probabilità, valutata soggettivamente, di conseguire un ritorno pari a ΔA_{t-1} (più alto è r_s, minore è la propensione all'investimento innovativo o più esagerata la valutazione del rischio); $(1 + r)^{-\lambda}$ è un fattore di attualizzazione finanziaria al tasso di mercato r che tiene conto dello sfasamento temporale tra investimento ΔA_t e ritorno atteso ΔK_t; λ è dimensionato all'ampiezza del periodo t.

La dinamica di crescita dell'innovazione. – Risolvendo l'equazione di innovazione (1.5) risulta, come detto in precedenza:

$$\varepsilon^{\delta} = (\alpha/r)^{\frac{1}{1-\alpha}} \zeta = f(r, r_s, \lambda)$$

Dunque la funzione di innovazione $A_t = A_0 \varepsilon^{\delta t}$ (funzione discreta di t) ha una dinamica (ε^{δ}) che dipende endogenamente dai principali meccanismi di mercato espressi dalle equazioni del modello. Tale dinamica è in conclusione correlata (negativamente) col tasso r, con il tasso r_s e con la durata λ dell'intero circuito. Sui fattori che influenzano il livello del tasso r non ci soffermiamo in questa sede, l'argomento meriterebbe un'ampia trattazione, che esula dagli scopi del presente lavoro; sul tasso r_s, uno degli elementi centrali della nostra analisi, facciamo invece le seguenti considerazioni: r_s è un elemento caratteristico, assimilabile per taluni aspetti a « *factor endowment* », di ogni sistema: r_s elevato indica bassa propensione all'investimento innovativo.

Nel livello di r_s si riflettono, oltre che le attitudini, le aspettative dell'imprenditore; queste a loro volta riflettono l'immagine che egli ha delle principali variabili economiche ed extraeconomiche del settore e del sistema in cui opera. Sicché in r_s si riflettono gli elementi essenziali dell'intero sistema, sia pure attraverso la percezione soggettiva degli imprenditori.

La diffusione. – Come accennato, i continui reinvestimenti del profitto generano una continua espansione di K che riduce il tasso di profitto *schumpeteriano* π fino ad annullarlo; quando $\pi = 0$, la diffusione della nuova tecnologia è totale, essa è nota a tutti (*public knowledge*) e il prodotto è vendibile solo in condizioni di concorrenza.

La diffusione dell'innovazione fino alla saturazione dello specifico settore (o fino al superamento dell'innovazione con altra che la renda obsoleta) può avvenire ad opera del solo imprenditore innovatore o più spesso con il concorso di altri imprenditori imitatori. Nel secondo caso la somma degli utili complessivi di diffusione non varia, mentre cambia l'ammontare di utili conseguiti (*ex post*) da ciascun imprenditore. In ogni caso la stima *ex ante*, per ipotesi frutto di una valutazione soggettiva, errata o esatta, ha già determinato la misura di ΔA_t.

Grandezze caratteristiche nei punti estremi del periodo t. – Da $K_t = (\alpha/r)^{\frac{1}{1-\alpha}} A_t$ si ricava $\Delta K_t = (\alpha/r)^{\frac{1}{1-\alpha}} \Delta A_t$. Confrontando ΔK_t con ΔA_t risulta $\Delta K_t/\Delta A_t = (\alpha/r)^{\frac{1}{1-\alpha}}$. Il rapporto, generalmente maggiore di 1, ha una correlazione inversa con r, il che vuol dire che, fermo ΔA_t, la diffusione ΔK_t è negativamente influenzata da r.

Essendo $Y_t = A_t^{1-\alpha} K_t^{\alpha}$, risulta, nei punti estremi dei periodi t, $Y_t = (\alpha/r)^{\frac{\alpha}{1-\alpha}} A_t$.

La lezione schumpeteriana. – Le dinamiche descritte raffigurano un fenomeno focalizzato da Schumpeter e riconoscibile, in modo a volte più evidente a volte meno, nella realtà empirica: a un'innovazione segue di solito una serie di applicazioni realizzate mediante investimenti materiali e immateriali, sempre meno remunerativi a mano a mano che il *know-how* si diffonde e il mercato del bene si satura. Tutto ciò naturalmente in un mercato in concorrenza [12].

Ulteriori precisazioni. – Nel confrontare le dinamiche esposte con la realtà concreta sono opportune alcune ulteriori precisazioni. L'impresa innovativa in esame non gestisce per ipotesi un monopolio, essa gode però di un *know-how* esclusivo per un periodo limitato nel tempo, come spesso avviene nella realtà.

Nella schematizzazione del modello viene sviluppato un solo circuito innovazione-diffusione per volta; la schematizzazione, funzionale a una visione ai raggi X della dinamica in esame, la rende astratta e irreale se riferita all'intero sistema; nella realtà si verificano simultaneamente numerosi processi del tipo illustrato sfasati e accavallantisi.

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[12] Nel descrivere questa dinamica il modello adotta delle semplificazioni, alcune delle quali tipiche dei modelli marginalisti, come l'ipotesi classica che le variabili W e R siano in ogni istante in equilibrio *walrasiano* con le rispettive produttività marginali, mentre queste si muovono dietro la spinta di A_t, che si trasmette a K_t, e quindi a Y_t e a tutto il sistema.

È possibile inoltre che l'innovazione sia autofinanziata e la diffusione sia finanziata con prestiti esterni. Tutto ciò non annulla il fenomeno che il modello intende evidenziare: ogni investimento innovativo di successo ha un ritorno complessivo superiore al suo costo finanziario; il *surplus* così prodotto ha solitamente una dimensione finita e concorre all'autofinanziamento degli investimenti.

Ad attivare il processo in questione, e in definitiva a promuovere la crescita, è l'imprenditore (la funzione impresa) in un mercato competitivo, che utilizza strumenti finanziari diversi a seconda delle circostanze e del contesto, ma è distinto (come ruolo) dal finanziatore.

3. *Il sentiero di crescita d'equilibrio e la distribuzione*

È utile ai nostri fini rettificare l'ipotesi secondo la quale in equilibrio di concorrenza, cioè con tecnologie totalmente note e diffuse, vale la relazione $\partial Y/\partial K - r = \pi = 0$.

Nella realtà un imprenditore non correrebbe i rischi dell'impresa per un compenso pari al tasso d'interesse dei prestiti sui capitali impiegati, compenso comunque ottenibile senza assumere i rischi d'impresa. È più realistico pensare che in equilibrio di concorrenza con tecnologie totalmente note e diffuse valga la seguente relazione:

$$\partial Y_t/\partial K_t = \mu + r = r' \qquad (\mu > 0) \tag{3.1}$$

Nei punti estremi di ogni intervallo temporale valgono le seguenti relazioni d'equilibrio *ex post*:

$$K_t = (\alpha/r')^{\frac{1}{1-\alpha}} A_t \tag{3.2}$$

$$Y_t = (\alpha/r')^{\frac{\alpha}{1-\alpha}} A_t \tag{3.3}$$

Le funzioni discrete K_t e Y_t hanno il medesimo tasso di crescita annuo di $A_t = A_0 e^{\delta t}$, cioè:

$$\Delta/A_t/A_t = \Delta K_t/K_t = \Delta Y_t/Y_t = e^{\delta} - 1 = i \text{ (per brevità).}$$

Ne conseguono i seguenti rapporti periodali (*ex post*):

$$\Delta A_t/Y_t = i\,(r'/\alpha)^{\frac{\alpha}{1-\alpha}} \tag{3.4}$$

$$\Delta K_t / Y_t = i\,\alpha / r' \tag{3.5}$$

$$\Delta A_t / \Delta K_t = (r'/\alpha)^{\frac{1}{1-\alpha}} \tag{3.6}$$

Per valori significativi dei parametri r', α

$$\Delta A_t < \Delta K_t$$

Il consumo del periodo è così definibile:

$$C_t = Y_t - (\Delta A_t + \Delta K_t) = \{(\alpha/r')^{\frac{\alpha}{1-\alpha}} - i - i\,(\alpha/r')^{\frac{1}{1-\alpha}}\}\,A_t \tag{3.7}$$

dunque anche il consumo cresce al medesimo tasso di A_t.

La forma scelta per la funzione di produzione ($Y = A^{1-\alpha} K^{\alpha} L^{\beta} T^{\gamma}$), e in particolare per $A^{1-\alpha}$, è funzionale alle ipotesi del modello e in particolare alla relazione dinamica fra A_t e K_t; essa garantisce una relazione sistematica e costante degli investimenti diffusivi ΔK_t rispetto agli investimenti innovativi ΔA_t, come è illustrato più in dettaglio in Appendice.

Il rapporto fra investimenti ($\Delta A_t + \Delta K_t$) e prodotto (Y_t) è costante rispetto a t ed è tanto più alto quanto più alto è δ:

$$(\Delta A_t + \Delta K_t)/Y_t = i\,[(r'/\alpha)^{\frac{\alpha}{1-\alpha}} + \alpha/r'] \tag{3.8}$$

È costante anche il rapporto

$$C_t/Y_t = [Y_t - (\Delta A_t + \Delta K_t)]/Y_t = 1 - i\,[(r'/\alpha)^{\frac{\alpha}{1-\alpha}} + \alpha/r'] \tag{3.9}$$ [13]

tanto più basso quanto più è alto δ.

Dalle relazioni d'equilibrio di fine periodo

$$\partial Y_t / \partial L = \beta\, A_t^{1-\alpha} K_t^{\alpha} L^{\beta-1} T^{\gamma} = W_t \tag{1.1}$$

$$\partial Y_t / \partial T = \gamma\, A_t^{1-\alpha} K_t^{\alpha} L^{\beta} T^{\gamma-1} = R_t \tag{1.2}$$

si ricava:

[13] Si ricorda che $i = e^{\delta} - 1$.

$$(\beta/L)\, Y_t = W_t \tag{1.1''}$$

$$(\gamma/T)\, Y_t = R_t \tag{1.2''}$$

da cui:

$$\Delta W_t/W_t = \Delta Y_t/Y_t = i$$

$$\Delta R_t/R_t = \Delta Y_t/Y_t = i$$

I tassi annui di crescita delle remenerazioni unitarie d'equilibrio W_t, R_t sono uguali a quelli di Y_t, K_t, A_t, C_t, se, come ipotizzato, i livelli massimi di L e T restano costanti nel tempo.

Riepilogando, alla crescita della produttività A_t corrisponde una uguale crescita della produzione Y_t, del consumo C_t e del fattore espandibile K_t, parallela alla crescita delle remunerazioni W_t, R_t dei fattori non espandibili L, T, se r è costante.

Il tasso di crescita effettivo annuo $e^{\delta} - 1 = i$[14] $= (\alpha/r)^{\frac{1}{1-\alpha}} (1 + r)^{-\lambda} (1 + r_s)^{-\lambda} - 1$ è negativamente influenzato dal tasso di mercato dei prestiti r, e dal tasso di valutazione del rischio r_s. Se questi due tassi sono elevati è inoltre maggiore la propensione al consumo C_t/Y_t e minore la propensione all'investimento I_t/Y_t.

La distribuzione. – Nei punti estremi degli intervalli temporali, cioè in equilibrio di concorrenza di piena occupazione, il prodotto Y è interamente distribuito fra i 3 fattori K, L, T[15]. Nel corso del periodo t, cioè nel corso del circuito *schumpeteriano*, un quarto fattore partecipa alla distribuzione: l'imprenditore. Questi, indipendentemente dall'eventuale possesso del capitale K (per chiarezza espositiva ipotizziamo che l'imprenditore non possieda capitale proprio inizialmente), percepisce un reddito (πK) distinto da quello di capitale (rK), come illustrato in precedenza. Per ipotesi del modello egli reimpiega tutti i profitti in capitale diffusivo K_t, realizzando un investimento diffusivo globale di periodo ΔK_t che costituisce per altro verso capitale di cui egli è titolare. Avendo ricevuto in prestito dal sistema finanziario ΔA_t iniziale, egli deve restituirlo con gli interessi; per far ciò utilizza una parte di ΔK_t di cui è titolare.

Al termine del circuito *schumpeteriano* all'imprenditore resta dunque

[14] δ è il corrispondente tasso istantaneo.

[15] È facile verificare in base alle relazioni (1.1), (1.2.), (1.3.) che $W_t L + R_t T + r K_t = (\alpha + \beta + \gamma)\, Y_t = Y_t$.

una parte di ΔK_t, uguale a $\Delta K_t - \Delta A_t (1 + r)^\lambda$; ai finanziatori resta l'altra parte di ΔK_t uguale a $\Delta A_t (1 + r)^\lambda$ [16].

Se questo meccanismo si ripete in ogni circuito, al tempo t la titolarità di K_t sarà in parte dei finanziatori. In termini di « *assets* » si può ipotizzare che A_t abbia un valore pari a zero; infatti gli investimenti per la ricerca (i brevetti per esempio) non hanno più alcun valore patrimoniale una volta che l'invenzione sia totalmente nota (*public knowledge*), sebbene abbia concorso ad accrescere la produttività di tutti i fattori.

Così K_t contiene il valore patrimoniale al tempo t di tutti i beni capitali, la cui titolarità è divisa fra due classi: i finanziatori e gli innovatori, premiati entrambi per il loro distinto apporto.

Sebbene semplificate e schematiche, queste ipotesi raffigurano taluni aspetti delle dinamiche di accumulo dei sistemi, specie se innovativi e dinamici. Le dinamiche di accumulo, di cui quella illustrata è solo un esempio, svolgono un ruolo importante nei fenomeni economici.

Conclusioni

La visione della crescita con progresso tecnico incorporato proposta si distacca in più punti da quella degli altri modelli di ispirazione classica e neoclassica e da quella di Grossman in particolare. Fra tutti, i seguenti appaiono di particolare rilievo.

1) Vengono indagate dinamiche dal lato della produzione, spesso in ombra nei modelli classici e neoclassici. Al centro di esse viene collocata « la funzione impresa » totalmente distinta dalla funzione del detentore del capitale finanziario. Ciò riflette alcune tendenze osservabili in sistemi avanzati in cui le due funzioni risultano in molti casi (in misura maggiore o minore) distinte.

2) L'investimento è essenziale all'innovazione: l'intuizione imprenditoriale non si traduce in progetto concreto (o in prototipo) senza investimento nell'innovazione; la dinamica di crescita nel suo complesso non si realizza senza investimento innovativo prima e diffusivo poi [17], la distinzione fra accumulazione delle conoscenze tecniche e accumulazione di capitale viene eliminata in quanto l'accumulazione delle conoscenze è essa stessa accumulazione di capitale.

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[16] Abbiamo già verificato che $\Delta K_t > \Delta A_t$ per valori significativi dei parametri r, α.

[17] La tendenza qui evidenziata, pur frequente nei settori innovativi, non ha ovviamente un valore assoluto: non sempre l'innovazione è generata da significativi investimenti.

3) L'investimento è generato dal risparmio d'impresa oltre che dal ricorso al finanziamento esterno.

4) Il modello evidenzia la tendenza degli investimenti e della stessa produzione a smaterializzarsi in termini di valore nei settori innovativi.

Nella realtà economica i principi enunciati trovano frequente riscontro.

Si richiama in fine l'attenzione su di un aspetto teorico importante: come si è visto, nel modello sono presenti le dinamiche di accumulazione del capitale (materiale e immateriale) di origine *solowiana*, il progresso tecnico incorporato che consente il formarsi del profitto *schumpeteriano*, gli equilibri *walrasiani* fra prezzi dei fattori e produttività marginale e un importante elemento del *factor endowment* specifico di ciascun sistema: la propensione all'investimento innovativo (attraverso il tasso r_s). Dunque, un modello di tipo (*solowiano*) accumulativo non è incompatibile con l'incorporazione del progresso tecnico, né con il concetto di *factor endowment* specifico di ciascun sistema, né con gli equilibri *walrasiani*.

APPENDICE

Dall'equazione dinamica di diffusione (1.5) risulta, come abbiamo visto, la seguente relazione d'equilibrio (stabile in ogni estremo del periodo t) fra K_t e A_t: $K_t = (\alpha/r)^{\frac{1}{1-\alpha}} A_t$ da cui $Y_t = (\alpha/r)^{\frac{\alpha}{1-\alpha}} A_t$.

K_t reagisce alle variazioni di A_t, trascinando con sé Y_t; al termine della reazione i rapporti A_t/K_t, K_t/Y_t ritornano al loro valore costante in ogni t (in altri termini i rapporti anzidetti sono indipendenti da t).

Questo effetto, conforme agli obiettivi e alle ipotesi del modello, è ottenibile unicamente dando ad A l'esponente $1-\alpha$.

La funzione di produzione ha solitamente la forma: $Y = A\,K^{\alpha} L^{\beta}$, $(\alpha + \beta = 1)$, dove l'esponente di A è 1. Tale forma, convenzionale quanto quella proposta dal modello, renderebbe i rapporti anzidetti variabili in funzione di t, tradendo in tal modo uno degli obiettivi principali del modello, che ipotizza una reazione non dipendente dal tempo storico degli investimenti diffusivi ΔK_t agli investimenti innovativi ΔA_t, come si osserva frequentemente nella realtà.

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SCHUMPETER'S DYNAMICS AND GROWTH

This paper analyzes growth dynamics following Romer and Grossman models of long-run growth with endogenous technological change. The model presented breaks away from those theories in certain important areas, in the wake of Schumpeter's thought.

Various production dynamics, often overshadowed by classical and neoclassical models are examined emphasizing the entrepeneur's role independently of the stock market. The role of cash-flow in financing investment is also examined.

Capital investment, completely distinct from knowledge growth, has a negligible influence in the Grossman model. In the present model on the contrary capital accumulation is systematically joined with the accumulation of knowledge which is an immaterial capital.

Finally, the author shows how, in innovative sectors, production tends to become immaterial (in value terms).

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ANTICIPATING A PRODUCTION QUOTA AND INVESTMENT

by
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I. *Introducing*

With economic policy rarely conceived in a vacuum, in most cases the means to implement the policy goal become subject of intense and public debate. Consequently, the impact of a commodity specific policy on an industry depends on whether the policy has been anticipated or not and how long the policy is expected to last. This paper focuses on how current investment behavior is influenced by the prospect of a future constraint on production – specifically, a production quota – to be introduced at an unknown future date. Production quotas have been traditionally employed as a supply control instrument in agriculture and natural resource management (e.g., peanuts and tobacco in the United States, milk in Canada and the European Community, fisheries in the U.S. and Canada).

Oftentimes, quotas are instituted in conjunction with public intervention prices. The choice of a production quota to address surplus production may be the path of least resistance for economic policy makers facing political pressures. The burden on the public coffers is eased since the quota no longer maintains an automatic commitment of the regulatory agency to purchase all production encouraged by a public intervention price. Further, with the removal of automatic supports it is now easier to discriminate between different categories or quality of production. With automatic supports set above the long-run market clearing price, a lower support price results in lower incomes to commodity producers. The political resistance to

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reduce income support to commodity producers is lessened since a lower quota does not necessarily imply lower profits.

Blanchard and Sachs (1982) and D'Autume and Michel (1985) focus on the investment strategy of the firm anticipating a future production control – specifically, a constraint on the quantity of capital goods the firm can acquire. Blanchard and Sachs find that firms engage in anticipatory buying of investment goods. However, D'Autume and Michel show that in the presence of a constant returns to scale technology the firm invests less preceding the constraint since the absence of an optimal scale does not require the firm to compensate in advance for the future deficit in investment spending. They conclude that with capital less valuable in the future the optimal rate of investment is reduced at earlier dates.

In order to separate risk effects from anticipation effects, a model of the risk neutral firm facing adjustment costs is developed characterizing investment behavior before and after a binding quota is introduced. The investment behavior for both firms anticipating and not anticipating a production quota is then characterized.

II. *Model of Investment Under a Quota*

Consider the firm facing the prospect of a production quota being introduced at some unknown future date, τ. Instantaneous cost is $C(K, y) + G(I) + qK$ where

$$C(K, y) = \min_{x} \, [w'x \,|\, y = f(x, K)]$$

where x is a vector of variable inputs with prices w, y is a single output, K is capital (the quasi-fixed factor), I is the rate of gross investment and q is the rental cost of capital. The production function $f(\;)$ is assumed to be well-behaved and $G(I)$ represents adjustment costs with $G_I > 0$ for $I > 0$ and $G_{II} > 0$ to guarantee the sluggish accumulation of capital.

Once the quota is introduced the firm is assumed to take the quota parametrically. This is tantamount to the firm maintaining myopic expectations concerning the level of the quota [1]. The firm's post-quota objective is to choose its production level and rate of investment to

[1] Production quotas are often set according to the historical production base. However, if producers are suspected of trying to inflate the historical base immediately before the quota is introduced, policy makers can select a historical base production period pre-dating the strategic production base-building period.

$$J^*(k_\tau, \bar{y}) = \max_{y, I} \int_\tau^\infty e^{-rs}[py - C(K, y) - G(I) - qK]\, ds \tag{1}$$

subject to

$$\dot{K} = I - \delta K \qquad K(\tau) = k_\tau \tag{2}$$

$$y \leq \bar{y} \tag{3}$$

where "·" indicates the time derivative, $\bar{y}$ is the quota on output, r and δ are the constant rates of discount and depreciation, respectively, and the value function J^* is valued in terms of the starting time τ. Assuming J^* is sufficiently differentiable the Hamilton-Jacobi equation is

$$rJ^* = \max_{y, I, \mu}[py - C(k_\tau, y) - G(I) - qk_\tau + (I - \delta k_\tau) J^*_{k_\tau} + \mu(\bar{y} - y)] \tag{4}$$

where μ is a Lagrangian multiplier associated with the quota constraint. The first order conditions are

$$p = C_y(k_\tau, y^*) + \mu^* \tag{5}$$

$$\mu^*(\bar{y} - y) = 0, \qquad \mu^* \geq 0 \tag{6}$$

$$G_I(I^*) = J^*_{k_\tau} \tag{7}$$

where * indicates the optimized value. Equation (5) accounts for the presence of a binding quota and states that the output price equals the instantaneous marginal cost of production plus the Lagrangian which is interpreted below. Equation (6) is the complementary slackness condition from Kuhn-Tucker theory. Equation (7) states that the marginal cost of adjustment equals the shadow cost of capital.

Differentiating both sides of the optimized Hamilton-Jacobi equation in (4) with respect to $\bar{y}$ yields

$$rJ^*_{\bar{y}} = \dot{K}^* J^*_{k_\tau \bar{y}} + \mu^*$$

Differentiating $J^*_{\bar{y}}(k_\tau, \bar{y})$ with respect to time (holding $d\bar{y} = 0$) yields

$$\frac{d(J_{\bar{y}})}{dt} = J^*_{\bar{y}k_\tau} \dot{K}^*$$

Thus,

$$\mu^* = rJ^*_{\bar{y}} - \frac{d(J^*_{\bar{y}})}{dt} \tag{8}$$

stating the Lagrangian is the opportunity cost of an increase in the value function arising from an increase in the quota less the instantaneous change in the marginal value of the quota. A marginal relaxation of a binding constraint on production should serve to enhance (or not further reduce) the expected net present value of the production plan leading to $J^*_{\bar{y}} \geq 0$. Further, the shadow value of capital should increase (or not decrease) with a relaxation of the quota since the capital stock's use is no longer as restricted leading to $J_{k_\tau \bar{y}} \geq 0$. To see this, differentiate both sides of (7) with respect to $\bar{y}$ yielding

$$J_{k_\tau \bar{y}} = G_{II} \frac{\partial I^*}{\partial \bar{y}}$$

Convex adjustment costs imply $G_{II} > 0$. If the quota is not binding, optimal investment is not influenced by an increase (or relaxation) of the quota. If the quota is binding, the optimization is equivalent to minimizing cost. Assuming investment is input increasing in the output target (i.e., a normal input), $\partial I^* / \partial \bar{y} > 0$, implying $J_{k_\tau \bar{y}} \geq 0$. With $\mu^* \geq 0$, $rJ^*_{\bar{y}} \geq \dot{K}^* J^*_{k_\tau \bar{y}}$.

Pre-Quota Investment Behavior: Known Switching Date. – In advance of the introduction of the perfectly anticipated quota, the firm's intertemporal maximization problem is

$$L(k_t, \bar{y}) = \max_{y,I} \int_t^\infty e^{-rs} [py - C(K, y) - G(I) - qK] \, ds \tag{9}$$

subject to (2) with $K(t) = k_t$ in advance of time τ. The value function L is valued in terms of the starting time t. Appealing to the principle of optimality, (9) is rewritten as

$$L(k_\tau, \bar{y}) = \max_{y,I} \int_t^\tau e^{-rs} [py - C(K, y) - G(I) - qK] \, ds$$

$$+ e^{-r(\tau - t)} \int_\tau^\infty e^{-rs} [py - C(K, y) - G(I) - qK] \, ds$$

$$= \max_{y,I} \int_t^t e^{-rs} [py \; C(K, y) - G(I) - qK] \, ds + e^{-r(\tau - t)} J^*(k_\tau, \bar{y}) \tag{10}$$

subject to $y(s) \leq \bar{y}$ when $s \geq \tau$. $J^*(k_\tau, \bar{y})$ is defined in (1) and can be viewed as a terminal function which is alternatively expressed as

$$J^*(k_\tau, \bar{y}) = J(k_t, \bar{y}) + \int_t^\tau \frac{dJ^*}{ds} ds = J^*(k_t, \bar{y}) + \int_t^\tau \dot{K} J_k^* ds \quad (11)$$

Using (11) in (10) leads to

$$L(k_t, \bar{y}) = e^{-r(\tau-t)} J^*(k_t, \bar{y})$$

$$+ \max_{y,I} \int_t^\tau e^{-rs} [py - C(K, y) - G(I) - qk + e^{-r(\tau-s)} \dot{K} J_k^*] ds$$

where $J^*(k_t, \bar{y})$ is a starting function which is constant and $e^{-r(\tau-s)} \dot{K}^* J_k^*$ reflects the impact of changes in capital accumulation at time $s < \tau$ on the value function starting at time τ discounted back to s.

The Hamilton-Jacobi equation at base time period t is

$$rL(k_t, \bar{y}) = e^{-r(\tau-t)} J^*(k_t, \bar{y}) + \max_{y,I} [py - C(k_t, y) - G(I) - qk_t$$

$$+ (I - \delta k_t) \{e^{-r(\tau-t)} J_k^*(k_t, \bar{y}) + L_k(k_t, \bar{y})\}]$$

Let α denote the firm facing a quota at known time τ and β denote the firm not facing a quota. Assuming L is sufficiently differentiable and both firms start with the same capital stock (k_t) and employ the same production technology, the first order conditions for firms α and β at $t < \tau$ are

$$p = C_y(k_t, y^*) \text{ for both } \alpha \text{ and } \beta \quad (12.a)$$

$$G_I(I^\alpha) = e^{-r(\tau-t)} J_k^{*\alpha}(k_t, \bar{y}) + L_k^\alpha(k_t, \bar{y}) \quad (12.b)$$

$$G_I(I^\beta) = e^{-r(\tau-t)} J_k^{*\beta}(k_t) + L_k^\beta(K_t) \quad (12.c)$$

where y^* is the optimal production level. The marginal value of capital in the current period is described as the marginal value of capital looking from the current period to time τ, L_k, plus the marginal value of capital looking from time τ to the end of the planning horizon, $e^{-r(\tau-t)} J_k^*$. In addition, (12.b) indicates that as the starting time t approaches the switching date τ, $L_K^\alpha = J_k^{*\alpha}$, which is the transversality condition (Kamien and Schwartz, 1991, p. 160).

If

$$e^{-r(\tau-t)} J_k^{*\alpha}(k_t, \bar{y}) + L_k^{\alpha}(K_t, \bar{y}) \gtreqless e^{-r(\tau-t)} J_k^{*\beta}(k_t) + L_k^{\beta}(K_t)$$

then $G_I(I^\alpha) \gtreqless G_I(I^\beta)$. Since the marginal adjustment cost is increasing in investment, the firm with the higher G_I invests more in the current period. Therefore, $G_I(I^\alpha) \gtreqless G_I(I^\beta)$ implying $I^\alpha \gtreqless I^\beta$. The presence of a binding constraint on the production level does not increase the value of an additional unit of capital leading to $J_k^{*\beta}(k_t) \geq J_k^{*\alpha}(k_t, \bar{y})$ and $L_k^{\beta}(k_t) \geq L_k^{\alpha}(k_t, \bar{y})$ with strict equality holding for $\bar{y} = y^*$. Consequently,

$$e^{-r(\tau-t)} J_k^{*\alpha}(k_t, \bar{y}) + L_k^{\alpha}(k_t, \bar{y}) \leq e^{-r(\tau-t)} J_k^{*\beta}(k_t) + L_k^{\beta}(k_t)$$

implying $I^\alpha \leq I^\beta$. Thus, current investment for the firm facing a production quota at a known future date does not exceed the current investment of the firm not facing a quota.

Pre-Quota Investment Behavior: Unknown Switching Date. – In advance of the introduction of the stochastically anticipated quota, the firm's intertemporal profit maximization problem can be expressed as

$$V(k_t, \bar{y}) = \max_{y,I} E_{\tau|t} \left\{ \int_t^\tau e^{-rs} [py - C(K, y) - G(I) - qK] \, ds + e^{-r(\tau-t)} J^*(k_\tau, \bar{y}) \right\} \quad (13)$$

subject to (2) with $K(t) = k_t$, (3) and $t < \tau$ where $E_{\tau|t}$ denotes the expectation over the random variable τ starting at time t. Let $\phi(\tau)$ be the probability density function of the quota introduction date, τ, where τ ranges from (t, ∞). The problem in (13) can be simplified by carrying out the expectation and integrating by parts to yield

$$V(k_t, \bar{y}) = \max_{y,I} \int_t^\infty e^{-rs} \{[py - C(K, y) - G(I) - qK] \, \Phi(\tau) + J^*(K, \bar{y}) \, \phi(\tau)\} \, d\tau \quad (14)$$

where $\Phi(\tau) = \int_\tau^\infty \phi(s) \, ds$. In general, the optimization problem in (14) results in a nonautonomous system. Following Yaari (1965), Dasgupta and

Heal (1974), and Blanchard (1985) who exploit the exponential probability density function leading to an autonomous system, assume $\phi(\tau) = \lambda e^{-\lambda\tau}, \lambda > 0, \tau > t$. The optimization problem simplifies to

$$V(k_t, \bar{y}) = \max_{y,I} \int_t^{\infty} e^{-(r+\lambda)s} [py - C(K, y) - G(I) - qK + \lambda J^* e^{-r(\tau - t)}] \, ds$$

where λ can be interpreted as the ratio of the probability the quota is introduced today to the probability it is introduced at a later date; or, alternatively, λ^{-1} is the expected time before the quota is introduced. λ augments the discounting factor increasing the opportunity cost of investment decisions.

The Hamilton-Jacobi equation at the base time period is

$$(r+\lambda) V = \max_{y,I} [py - C(k_t, y) - G(I) - qk_t + \lambda J^*(k_t, \bar{y}) e^{-r(\tau - t)} + (I - \delta k_t) V_{k_t}]$$

Assume V is sufficiently differentiable and $\bar{y}$ is fixed, the first order conditions are

$$(p - C_y) = 0 \tag{15}$$

$$G_I = V_k \tag{16}$$

Inserting the optimized values of y and I into (10) and differentiating both sides with respect to k_t yields

$$(r + \delta + \lambda) V_k = -(C_k + q) + \dot{K}^* V_{kk} + \lambda J_k^* e^{-r(\tau - t)}$$

Differentiating $V_k(k_t, \bar{y})$ with respect to time (holding $d\bar{y} = 0$) yields

$$\frac{dV_k}{dt} = V_{kk} \dot{K}^*$$

Thus, the shadow value of capital anticipating the introduction of a production quota is

$$V_k = \frac{1}{(r+\delta+\lambda)}\left[-(C_k+q)+\lambda J_k^* e^{-r(\tau-t)}+\frac{dV_k}{dt}\right] \tag{17}$$

which is the arbitrage equation.

With the quota strictly binding, $y^* > \bar{y}$, define $\gamma(k_p, \bar{y})$ as the difference between the marginal adjustment costs when the quota is binding presently and when the quota is not binding,

$$\gamma(k_p, \bar{y}) = G_{I_{\bar{y}<y^*}} - G_{I_{\bar{y}\geq y^*}} \tag{18.a}$$

In advance of the introduction of the quota, $G_{I_{\bar{y}\geq y^*}} = V_k$, by (15). The problem in (1) reflects the case where the quota is presently binding, implying the first order condition in (7). Thus,

$$\gamma(k_p, \bar{y}) = J_k^*(k_p, \bar{y}) - V_k^*(k_p, \bar{y}) \tag{18.b}$$

The function $\gamma(\cdot)$ can be alternatively viewed as the difference between the shadow value of capital for a presently binding quota and when the quota is not binding presently. Given the shadow value of capital is increasing in y, $\gamma(\cdot)$ reflects the change in the shadow value of capital given a decrease in output (i.e., a sufficient decrease to force a binding quota). Therefore, it is expected that $\gamma(\cdot) < 0$.

Using (18.b) in (17) leads to

$$(r+\delta+\lambda)V_k = -(C_k+q)+\lambda\gamma(k_p,\bar{y})+\lambda V_k+\frac{dV_k}{dt}$$

$$(r+\delta)V_k = -(C_k+q)+\lambda\gamma(k_p,\bar{y})+\frac{dV_k}{dt} \tag{19}$$

or

$$(r+\delta)V_k < -(C_k+q)+\frac{dV_k}{dt}$$

Thus, the opportunity cost of an additional unit of capital is less than the traditional components of a change in the instantaneous cost, $-(C_k+q)$, plus the instantaneous capital gain (or loss), $\frac{dV_k}{dt}$. The difference is $\lambda\gamma(\cdot)$ reflecting the impact of a binding production in the future on the shadow value of capital weighted by λ.

III. *Anticipating Versus Nonanticipating Investment Behavior*

Let the optimal functions for the firm anticipating and not anticipating the quota be denoted by the superscripts α' and β, respectively. Equation (19) is $V_k^{\alpha'}$ and

$$(r + \delta) V_k^{\beta} = -[C_k(k_t, y^{\beta}) + q] + \frac{dV_k^{\beta}}{dt} \tag{20}$$

The first order conditions regarding output for firms α' and β are identical leading to $y^{\alpha'} = y^{\beta} = y^*$. Implicitly the optimal investment levels, $I^{\alpha'}$ and I^{β}, are functions of the endogenously determined shadow values, $V_k^{\alpha'}$ and V_k^{β}, respectively. Differentiating both sides of the first order conditions $G_I[I^{\alpha'}(V_k^{\alpha'})] = V_k^{\alpha'}$ and $G_I[I^{\beta}(V_k^{\beta})] = V_k^{\beta}$ with respect to their respective shadow values of capital leads to

$$\frac{\partial I^{\alpha'}}{\partial V_k^{\alpha'}} = \frac{\partial I^{\beta}}{\partial V_k^{\beta}} = \frac{1}{G_{II}} \tag{21}$$

Since $G(I)$ is independent of k_t (or more simply, if G_{II} is positive and a constant), this leads to

$$I^{\alpha'}(V_k^{\alpha'}) = z^{\alpha'} + G_{II}^{-1} V_k^{\alpha'} \tag{22}$$

$$I^{\beta}(V_k^{\beta}) = z^{\beta} + G_{II}^{-1} V_k^{\beta} \tag{23}$$

where

$$z^{\alpha'} = \delta\bar{k}^{\alpha'} - G_{II}^{-1} V_k^{\alpha'}(\bar{k}^{\alpha'})$$
$$z^{\beta} = \delta\bar{k}^{\beta} - G_{II}^{-1} V_k^{\beta}(\bar{k}^{\beta}) \tag{24}$$

and $\bar{k}^j$ is the steady state capital stock for firm j, $j = \alpha', \beta$.

A phase diagram can be used to illustrate how current investment differs for firms α' and β [2]. The $\frac{dV_k^j}{dt} = 0$ isoclines, $j = \alpha', \beta$, are defined as

[2] The relationship between $V_k^{\alpha'}$ and V_k^{β} can be determined by solving for $(C_k + q)$ in (20) and inserting the resulting expression into (19) to yield

$$V_k^{\alpha'} = V_k^{\beta} - \frac{1}{r+\delta}\left\{\frac{dV_k^{\alpha'}}{dt} - \frac{dV_k^{\beta}}{dt}\right\} + \frac{\lambda\gamma(k_t, \bar{y})}{r+\delta}$$

$$V_k^{\alpha'} = \frac{-(C_k + q) + \lambda\gamma(k_t, \bar{y})}{r+\delta}$$

$$V_k^{\beta} = \frac{-(C_k + q)}{r+\delta}$$

The slopes of these isoclines are

$$\frac{dV_k^{\alpha'}}{dk}\Big|_{\frac{dV_k^{\alpha'}}{dt}=0} = \frac{-C_{kk} + \lambda\gamma_k(\cdot)}{r+\delta}$$

$$\frac{dV_k^{\beta}}{dk}\Big|_{\frac{dV_k^{\beta}}{dt}=0} = \frac{-C_{kk}}{r+\delta}$$

Both firms follow the same first order condition for selecting output implying the same output level and start with the same capital stock leading to $(C_k + q)$ and C_{kk} the same for both firms α' and β. With $\lambda\gamma(\ ) < 0$, the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline initially will lie below the $\frac{dV_k^{\beta}}{dt} = 0$ isocline. The slope of the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline differs by the sign and magnitude of $\lambda\gamma_k(\ )$, where

$$\gamma_k(k_t, \bar{y}) = V_{kk}^{\alpha} - J_{kk}^{*}$$

With $V^{\alpha'}$ and J^* concave in k (a sufficient property for both value functions), the sign of γ_k is indeterminant. With $\gamma_k > 0$ (< 0) the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline is flatter (steeper) than the $\frac{dV_k^{\beta}}{dt} = 0$ isocline. The magnitude of the relative slopes of the isoclines is influenced by the magnitude of λ. With $\gamma_k > 0$, the larger λ, the more likely the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline crosses the $\frac{dV_k^{\beta}}{dt} = 0$ isocline.

The $\dot{K}^j = 0$ isocline implies $I^j(V_k^j) = \delta k_t$ with slope

$$\frac{dV_k^j}{dk}\Big|_{\dot{K}^j=0} = \frac{\delta}{\frac{\partial I^j}{\partial V_k^j}} = \delta G_{II}$$

for $j = \alpha', \beta$. Thus, both firms face the same $\dot{K} = 0$ isocline.

Three cases emerge. Figure 1 compares the current investment rate for

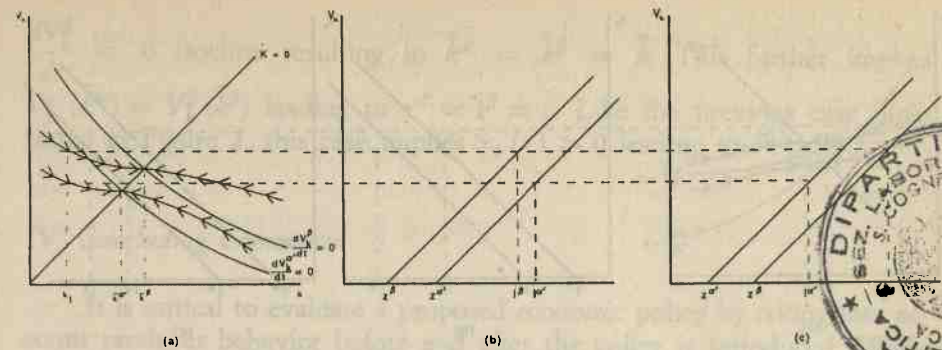

FIGURE 1.

firms α' and β when the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline lies below the $\frac{dV_k^{\beta}}{dt} = 0$ isocline. Panels 1.b and 1.c illustrate current investment using (22) and (23). From (24),

$$z^{\alpha'} - z^{\beta} = \delta\,(\bar{k}^{\alpha'} - \bar{k}^{\beta}) - G_{II}^{-1}\,[V_k^{\alpha'}\,(\bar{k}^{\alpha'}) - V_k^{\beta}\,(\bar{k})]$$

Since $\bar{k}^{\alpha'} < \bar{k}^{\beta}$ and $V_k^{\alpha'}\,(\bar{k}^{\alpha'}) < V_k^{\beta}\,(\bar{k}^{\beta})$, the sign of $(z^{\alpha'} - z^{\beta})$ is indeterminate. Panels 1.b and 1.c illustrate the cases of $z^{\alpha'} > z^{\beta}$ and $z^{\alpha'} < z^{\beta}$, respectively. One observes that,

$$z^{\alpha'} \gtreqless z^{\beta} \Rightarrow I^{\alpha'} \gtreqless I^{\beta}$$

The magnitude of λ influences the sign and magnitude of $(z^{\alpha'} - z^{\beta})$. If λ is large (small), then $V_k^{\alpha'}\,(\bar{k}^{\alpha'}) < (>)\, V_k^{\beta}\,(\bar{k}^{\beta})$. If $\bar{k}^{\alpha'} < \bar{k}^{\beta}$ and λ is large, $z^{\alpha'} < z^{\beta}$ which is the case illustrated in panel 1.c. If $\bar{k}^{\alpha'} < \bar{k}^{\beta}$ and λ is small, $z^{\alpha'} - z^{\beta}$ cannot be signed *a priori*. A large value of λ implies the expected quota introduction date is sooner rather than later leading to the firm behaving more like the firm knowing exactly when the quota will be imposed.

Figure 2 compares the current investment rate for firms α' and β when the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline crosses the $\frac{dV_k^{\beta}}{dt} = 0$ isocline. This case implies $\gamma_k(\cdot) > 0$. Panels 2.b and 2.c indicate the current investment when $z^{\alpha'} > z^{\beta}$ and $z^{\alpha'} < z^{\beta}$, respectively. Since $\bar{k}^{\alpha'} > \bar{k}^{\beta}$ and $V_k^{\alpha'}\,(\bar{k}^{\alpha'}) > V_k^{\beta}\,(\bar{k}^{\beta})$, the sign of $(z^{\alpha'} - z^{\beta})$ is indeterminate. As in the previous case,

$$z^{\alpha'} \gtreqless z^{\beta} \Rightarrow I^{\alpha'} \gtreqless I^{\beta}$$

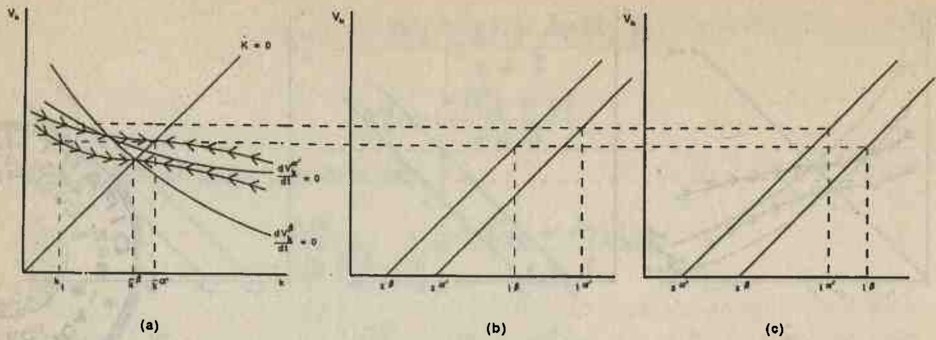

FIGURE 2.

With $\bar{k}^{\alpha'} > \bar{k}^{\beta}$ and λ small [implying $V_k^{\alpha'}(\bar{k}^{\alpha'}) < V_k^{\beta}(\bar{k}^{\beta})$], $z^{\alpha'} > z^{\beta}$ leading to $I^{\alpha'} > I^{\beta}$. This is the case illustrated in panel 2.c. λ small implies the expected quota introduction date is later rather than sooner leading the firm stochastically anticipating the quota to invest more than the firm not anticipating a quota. The uncertainty concerning the introduction date implies a higher steady state capital stock encouraging the firm to invest at a faster rate in the current period.

Figure 3 illustrates the case where the $\frac{dV_k^{\alpha'}}{dt} = 0$ isocline crosses the

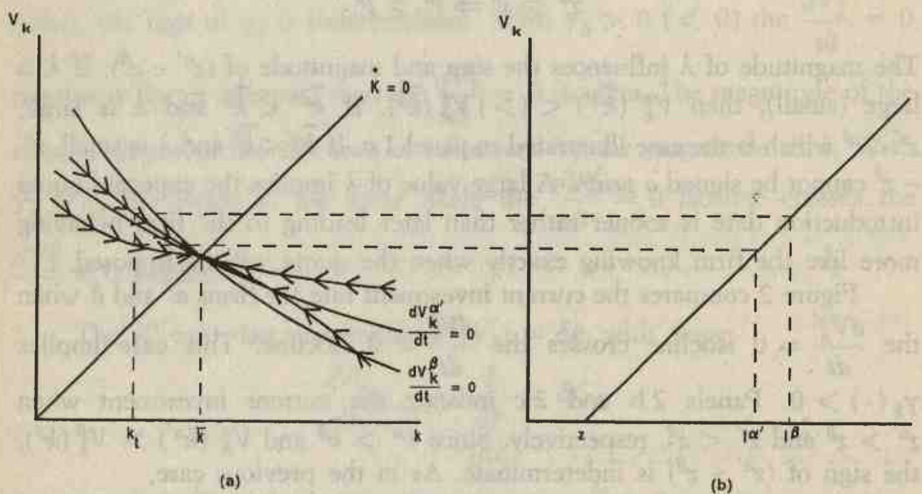

FIGURE 3.

$\frac{dV_k^\beta}{dt} = 0$ isocline resulting in $\bar{k}^{\alpha'} = \bar{k}^\beta = \bar{k}$. This further implies $V_k^{\alpha'}(\bar{k}^{\alpha'}) = V_k^\beta(\bar{k}^\beta)$ leading to $z^{\alpha'} = z^\beta = z$. Like the previous case illustrated in Figure 2, this case implies $\gamma_k(\cdot) > 0$ leading to $I^\beta > I^{\alpha'}$.

IV. *Concluding Comments*

It is critical to evaluate a proposed economic policy by taking into account producer behavior before and after the policy is introduced. While quotas are typically set on a historical production base, the anticipations of producers in advance of a quota can significantly influence the structure of production decision making once the quota is introduced. The muddled messages sent by policy makers as regulation is crafted are used by those who are the target of the regulation as imperfect information in their decision making. As the process for determining the policy instrument becomes long and protracted, realization of the policy objective can take longer than expected.

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PREVISIONE DI UNA QUOTA DI PRODUZIONE E DEGLI INVESTIMENTI

I cambiamenti di politica economica che sono oggetto di aperti dibattiti possono incoraggiare le imprese a eluderne gli obiettivi con la previsione dell'eventuale scelta dello strumento di politica economica. Questo articolo considera il comportamento d'investimento dell'impresa che prevede che sarà introdotta a una data futura ignota una quota di produzione. L'impresa che prevede una quota tende a investire meno (più) dell'impresa che non la prevede quando la probabilità che venga introdotta una quota nel periodo corrente è relativamente alta (bassa). Vengono discusse le condizioni perché si verifichi o non si verifichi la previsione.

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DYNAMICS OF INPUT-OUTPUT CHANGES

by
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Introduction

"Input-output" tables are the integral part of any traditional planning model. Although .nowadays "input-output" tables are not in use, in the developed countries after the immense interests on that subject during the "60"s and "70"s, the applications of "input-output" tables or their more enlarged version "Social Accounting Matrix" are widespread for the planning and policy analysis of the developing countries. However, at the same time statistical services of the developing countries are not strong so as to produce sufficient data to revise and update "input-output" tables with regular survey data. The delays in the production of "input-output" tables can seriously undermine the effective estimation of policy models for these countries. Thus there are needs for appropriate updating techniques for the "input-output" matrix which is practically feasible, i.e. the method which will not impose excessive demands on the statistical services.

The purpose of this paper is to analyse some of the existing techniques to update the "input-output" matrix and to suggest alternative techniques which will incorporate the "state-space" approach of the control system analysis in the updating process of the "input-output" matrices.

1. *Existing Methods of Updating "I-O" Matrices*

The best known method of updating a given I-O matrix is the "RAS" method where the matrix is adjusted according to the sums of the rows and sums of the columns of the matrix. (Stone, 1961; Lecomber, 1969).

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The basic idea is that the various determinants of the change in I-O coefficients due to technological changes, economics of scale and changes in relative price etc. can be summarised by biproportional relationships in which each industry will incorporate a pair of "substitution" and "fabrication" multipliers (r_i and s_j respectively) which will operate uniformly over the rows and columns of the I-O matrix. Suppose the base year I-O matrix is A_0. The projection year matrix A_t can be determined by the following relationship:

$$A_t = r A_0 s$$

where Y_t is the output vector in year t, U_t is the total intermediate output vector and V_t is the total intermediate input vector.

$$(A_t \hat{Y}_t)\, i = U_t$$

$$(A_t \hat{Y}_t)'\, i = V_t$$

where i is the unit summation vector (1, 1,..., 1) and $\hat{Y}_t$ is the diagonal matrix where the elements of the vector Y_t are placed in the leading diagonal with zeros elsewhere. The components "r" and "s" can be determined by minimising the function

$$\Sigma \left[A_{ij,\,t} \log \left(\frac{A_{ij,\,t}}{A_{ij,\,0}} \right) \right]$$

The above method was widely used in the literature (Johansen, 1968; Ghosh, 1964; Allen and Gossling, 1975; Bacharach, 1970) and in the UK Government statistical services. (See *Economic Trends*, Central Statistical Office, various issues). Almond (1970) has proposed an alternative method of estimation where we need to minimise $\Sigma\, (A_{ij,\,t} - A_{ij,\,0})^2$ with respect to the functional relationship

$$A_t = \hat{r}H + A_0 + H\hat{s}$$

$$H = ii'$$

Another alternative was suggested by Friedlander (1961) where we minimise

$$\Sigma \frac{(A_{ij,\,t} - A_{ij,\,0})}{A_{ij,\,0}} \qquad \text{subject to the functional relationship}$$

$$A_t = \hat{r}Y_0 = Y_0 = Y_0\hat{s}$$

where Y_0 is the output vector of the base year.

The initial requirements for the "RAS" method are the estimates of total input and total intermediate sale of each industry which are the row and column sums of the I-O matrix. Projections using the "RAS" method although computationally simpler than projecting each individual components by the time series method or regression analysis are not always reliable and perform no better than the original base year I-O matrix plus a statistical error term (Tilanus, 1966; Johansen, 1968). Lecomber (1969) showed that the projection obtained by the "RAS" method is equivalent to exponential projections of individual elements and hence involves an unacceptable upward bias. The estimates for U and V can be unreliable and difficult to obtain thus creating further problems regarding the reliability of the method. If complete matrices for several years are available "RAS" is an inefficient projection method, and an alternative method involving an econometric model to forecast individual elements can be developed.

Apart from the computational problems there are serious economic considerations behind the "RAS" techniques. The coefficients of the I-O matrix reflect the relative price structure, demand mix and output mix. If we project that I-O matrix without any consideration to these, the resultant I-O matrix may not reflect the true I-O matrix we want to identify. Given these problems it is essential to search for an alternative approach. In the next section, efforts are made to utilise the state-space approach to update the I-O matrix. The fundamental idea is to represent the I-O matrix as an integral part of a dynamic economic system. The basic idea is, if we accept that expectations regarding future values in addition to the past observed values can affect the estimates of the coefficients of the model in a significant way, then it is essential to update estimates of the coefficient of the model in the light of the changing expectations and changing information set.

2. *State-Space Approach and Kalman Filter*

The basic principle of Newton's dynamics is that the future evolution of a dynamic process is entirely determined by its present state. The behaviour of a dynamic system is represented by a system of ordinary differential equations. The differential equations are said to constitute a mathematical model of one system. In order to obtain a solution it is necessary to have a set of initial conditions which correspond to the physical quantitative needed to predict the future behaviour of the system. Thus the initial conditions and physical state variables are equal in number. In the state-space approach (Pontryagin et al. 1962) all the differential equations of a system are first order equations. The dynamic variables that appear in the system of first order equations are called the state variables. The variables that determine the behaviour of the state variables are called control variables. Suppose y is the state vector and u the control vector, then the system in the state-space can be defined as

$$\dot{y} = \frac{dy}{dt} = f(y, u, t) \qquad \text{or in a linear process}$$

$\dot{y} = \dfrac{dy}{dt} = A(t)\,y + B(t)\,u$ when $A(t)$ and $B(t)$ are matrices. Output of the system is represented by an output vector $x(t)$ which is a linear combination of the state and the control vectors, i.e.

$$x(t) = C(t)\,y(t) + D(t)\,u(t)$$

where C and D are matrices. This equation is called observation equation and $x(t)$ is called the observation vector. Suppose v and w are two normally distributed disturbance vectors and the system equation and the observation equation are respectively:

$$\dot{y} = Ay + Bu + Fv$$

$$x = Cy + w$$

then the optimum observer (or state estimator) for the state x is given by

$$\dot{\hat{y}} = A\hat{y} + bu + \hat{k}(x - C\hat{y})$$

where $\hat{k}$ is called the gain matrix which has to be chosen optimally. The optimum observer is called the Kalman filter (Kalman, 1960; Astrom, 1970). Given this basic idea, we can derive the appropriate filter, i.e. an optimal observer for an I-O matrix given the initial conditions regarding the expectation and variance of the I-O matrix and the output vector. Compared to the "RAS" method here we need more information about the I-O matrix and the movement of the output vector. The advantages are that we will be able to incorporate the dynamics of the I-O matrix and the output vector in the updating process of the I-O matrix. In this way it is possible for us to incorporate the flow of information from the past experiences and the future expectations regarding the economy into the dynamics of the "I-O" matrix.

3. *A Bayesian Filtering Method and the Updating Method*

In Section 2 the matrices describing the system A, B, F, C, are considered to be constant over time. If we relax that assumption and allow these matrices to move overtime we can obtain a time-varying system. If we can obtain their probability distribution we can analyse their stochastic characteristics. The updating method we are going to describe will exploit these assumptions and will try to characterise future matrices by deriving a dynamic relationship between the mean and variances (or covariances) of these matrices and their associated vectors.

We start with the assumption that the I-O matrix A_t is time-varying and stochastic and as a result its reduced form $(1 - A)^{-1}$ is also time varying and stochastic.

Suppose the economy can be represented by an input-output model

$$Y_t = A_t Y_t + X_t \tag{1}$$

where Y_t is the output vector, A_t is the input-output matrix, X_t is the final consumption vector.

We can re-write (1) as

$$Y_t = (I - A)_t^{-1} X_t = \pi_t X_t \tag{2}$$

Suppose we consider π (i.e. $(I - A)^{-1}$) matrix as time-varying and stochastic, satisfying the difference equation

$$\pi_{t+I} = \pi_t + \varepsilon_t \tag{3}$$

where ε_t is the noise associated with the input-output estimation and forecasting.

The associated input-output structure of the economy will be

$$Y_{t+1} = \pi_{t+1} X_{t+1} + W_{t+1} \tag{4}$$

(W_t are errors in forecasts and disturbances).

We have the following assumptions:

(a) Y_t, X_t can be measured exactly for all t ($t = 0, 1, \ldots N - 1$), N being the periods of observation in a time series.

(b) The state vector is normally distributed with finite covariance matrix.

(c) The noises ε_t and W_{t+1} are independent discrete white noises with known statistics i.e.

$$E(\varepsilon_t) = 0 = E(W_{t+1})$$

$$E(\varepsilon_t, \varepsilon_t') = Q\delta$$

$$E(W_t, W_t') = R\delta$$

where δ is the Kronecker delta and the above two covariance matrices are assumed to be positive definite.

(d) $P(\pi_{t+1} | \pi_t) = P(\varepsilon_t)$ and $(P(Y_{t+1} | \pi_{t+1}) = P(W_{t+1})$ are the conditional probability densities for π and Y; our problem is to evaluate:

$$E(\pi_{t+1} | Y^{t+1}) : = \pi^*_{t+1}$$

and

$$\text{cov}(\pi_{t+1} | Y_{t+1}) : = S_{t+1} \text{ (i.e. the error covariance matrix)}$$

where

$$Y^{t+1} = Y_1, Y_2, Y_3, \ldots Y_{t+1}$$

4. *Updating Method*

Given the assumptions of the previous section, we can obtain the conditional probability density function using Bayes rule, as

$$P(\pi_{t+1} | Y^{t+1}) \approx \int P(\pi_t | Y^t) P(\pi_{t+1} Y_{t+1} | \pi_t, Y^t) d\pi_t \tag{5}$$

where

$$P(\pi_t | Y^t) = \text{constant} \exp(- {}^1/_2 (\pi_t - \pi_t^*)^2 S_t^{-1}$$

$$\pi_t^* = \Delta\ E(\pi_t | Y_t)$$

$$S_t = \Delta\ \text{Cov}(\pi_t | Y^t)$$

where S_t is assumed to be invertible.

$$P(\pi_{t+1}, Y_{t+1} | \pi_t, Y^t)$$

$$= \text{constant} \exp\left\{- {}^1/_2 \begin{bmatrix} \pi_{t+1} & - \pi_t \\ Y_{t+1} - \Pi_{t+1}, & X_{t+1} \end{bmatrix}^2 C^{-1}\right\} \quad (6)$$

where

$$C = \begin{bmatrix} Q & O \\ O & R \end{bmatrix}, \qquad C^{-1} = \begin{bmatrix} Q_{-1} & 0 \\ 0 & R_{-1} \end{bmatrix}$$

Hence

$$P(\pi_{t+1}\, Y^{t+1}) = \text{constant} \int \exp(- {}^1/_2 J_t(1))\, d\pi_t \quad (7)$$

where

$$J_t(1) = (\pi_t - \pi_t^*)\, S_t^{-1}(\pi_t - \pi_t^*) + (\pi_{t+1} - \pi_t)'\, Q^{-1}(\pi_{t+1} - \pi_t)$$
$$+ (Y_{t+1} - \pi_{t+1} X_{t+1})'\, R^{-1}(Y_{t+1} - \pi_{t+1} X_{t+1}) \quad (8)$$

We can write (8) as

$$J_t(1) = (\pi_t - \pi_t^*)'\, S_t^{-1}(\pi_t - \pi_t^*) + \{(\pi_{t+1} - \pi_t^*) - (\pi_t - \pi_t^*)\}'.$$
$$Q^{-1}\{(\pi_{t+1} - \pi_t^* - (\pi_t - \pi_t^*)\} + \{(Y_{t+1} - \pi_t X_{t+1}) -$$
$$- (\pi_{t+1} - \pi_t^*)\, X_{t+1}\}' \quad R^{-1}$$
$$\{(Y_{t+1} - \pi_t^*\, X_{t+1}) - (\pi_{t+1} - \pi_t^*)\, X_{t+1}\} \quad (9)$$

Expanding (9) and after simplification we get

$$J_t(1) = \{(\pi_t - \pi_t^*) - L_t \quad Q^{-1}(\pi_{t+1} - \pi_t^*)\}'$$
$$L_t^{-1}\{(\pi_t - \pi_t^*) - L_t Q^{-1}(\pi_{t+1} - \pi_t^*)\} + (\pi_{t+1} - \pi_t^*)'$$
$$(Q^{-1} + X_{t+1} R^{-1} X_{t+1} - Q^{-1} L_t Q^{-1})(\pi_{t+1}^* - \pi_t^*) +$$
$$(Y_{t+1} - \pi_t^* X_{t+1})' R^{-1}(Y_{t+1} - \pi_t^* X_{t+1}) - 2(\pi_{t+1} - \pi_t^*)$$
$$X'_{t+1} R^{-1}(Y_{t+1} - \pi_t^* X_{t+1}) \quad (10)$$

where $L^{-1} := (S_t^{-1} + Q^{-1})$

Integration with respect to π_t yields

$$\text{constant} \int \exp(- 1/2\ J_t(1))\, d\pi_t = \text{constant} \exp(- 1/2\ J_t(2))$$

where

$$J_t(2) = (\pi_{t+1} - \pi_t^*)' (Q^{-1} - Q^{-1} L_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1})$$
$$(\pi_{t+1} - \pi_t^*) + (Y_{t+1} - \pi_t^* X_{t+1})' R^{-1} (Y_{t+1} - \pi_t^* X_{t+1}$$
$$- 2 (\pi_{t+1} - \pi_t^*)' X'_{t+1} R^{-1} (Y_{t+1} - \pi_t^* X_{t+1})$$

Hence

$$P(\pi_{t+1} | Y^{t+1}) = \text{constant} \exp (- 1/2 J_t(2))$$

Since $P(\pi_{t+1} | Y^{t+1})$ is proportional to the likelihood function, by maximising the conditional probability density function, we are also maximising the likelihood function, in order to determine π^*_{t+1}. Minimisation of $J_t(2)$ is equivalent to maximisation of $P(\pi^{t+1} | Y^{t+1})$. To minimise $J_t(2)$, we expand (10) and eliminating terms not containing π_{t+1} we obtain:

$$J_t(3) = \pi'_{t+1} (Q^{-1} - Q^{-1} L'_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1}) \pi_{t+1}$$
$$- 2\pi'_{t+1} (Q^{-1} - Q^{-1} L_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1}) \pi_t^*$$
$$- 2\pi'_{t+1} X'_{t+1} R^{-1} (Y_{t+1} - X_{t+1} \pi_t^*)$$

Differentiating with respect to π'_{t+1} and noting what matrix in the quadratic term is symmetric one obtains:

$$\frac{\partial J_{t(3)}}{\partial \pi'_{t+1}} = 2 (Q^{-1} - Q^{-1} L_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1}) \pi_{t+1}$$
$$- 2 (Q^{-1} - Q^{-1} L_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1}) \pi_t^*$$
$$- 2X'_{t+1} R^{-1} (Y_{t+1} - \pi_t^* X_{t+1}) \tag{11}$$

Equating to zero we get

$$\pi^*_{t+1} = \pi_t + (Q^{-1} - Q^{-1} L_t Q^{-1} + X'_{t+1} R^{-1} X_{t+1})^{-1}$$
$$X'_{t+1} R^{-1} (Y_{t+1} - \pi_t^* X_{t+1}) \tag{12}$$

Now consider the composite matrix

$$Q^{-1} - Q^{-1} L_t Q^{-1}, \text{ where } L_t := (S_t^{-1} + Q^{-1})^{-1}$$

Then we can write the above composite matrix as

$$Q^{-1} - Q^{-1}(S_t^{-1} + Q^{-1})^{-1} \tag{13}$$

According to the matrix identity of Householder (1953), which has the general form

$$(A + BCB') = A^{-1} - A^{-1}B(C^{-1} + B'A^{-1}B)B'A^{-1}$$

(13) can be written as

$$Q^{-1} - Q^{-1}(S_t^{-1} + Q^{-1})^{-1}Q^{-1} = (Q + S_t)^{-1} : = P_{t+1}^{-1} \tag{14}$$

Hence we can rewrite (12) as

$$\pi^*_{t+1} = \pi^*_t + K_{t+1}(Y_{t+1} - \pi^*_t X_{t+1}) \tag{15}$$

Where $K_{t+1} = S_{t+1} X'_{t+1} R^{-1}$

$$S_{t+1}^{-1} = (QS_t)^{-1} + X'_{t+1} R^{-1} X_{t+1}$$

It is possible using matrix identity, to write $S_t = (P_t^{-1} + X'_t R^{-1} X_t)^{-1}$ as

$$S_t = P_t - P_t X'_t (R + X_t P_t X'_t)^{-1} X_t P_t$$

Where $K_t = P_t X'_t (R + X'_t P_t X_t)^{-1}$

Thus given Q (variance of the input-output coefficients), S_t (error covariance matrix), R (variance of the output) and expected final demand X_{t+1} we can always determine (using (15) above) $(I - A)_{t+1}^{-1}$ given $(I - A)_t^{-1}$.

Because

$$P(\pi_{t+1} | Y^{t+1}) =$$
$$= \text{constant} \exp\{-1/2(\pi_{t+1} - \pi^*_{t+1})' S'_{t+1}(\pi_{t+1} - \pi^*_{t+1})\}$$

$P(\pi_{t+1} | Y^{t+1})$ is symmetric and unimodal about π^*_{t+1}, so all three best estimates i.e. conditional mean, median and mode of $P(\pi_{t+1} | Y^{t+1})$ are given by π^*_{t+1}.

The advantage of the above method is that we can take into account the error and disturbances that exist in the estimation of the model and in the estimation of the input-output matrix. Also we can prove that given the information set Y^{t+1} this updating method can produce the best estimates for $(I - A)^{-1}$ matrix of the period $t + 1$.

5. *Conclusion*

We have seen that although the "RAS" method demands minimum information it does not take into account the dynamic relationships of "I-O" coefficients among each other and their relationship with the output vectors over the past periods and with the demand vector in future. The state-space approach takes into account these complete relationships and as a result the dynamic of outputs, I-O coefficients and demand structure enter into the calculations. The updating method using the filtering approach takes into account both the past experiences and future expected values of the I-O coefficients, output and future demand. Thus rather than a mechanical updating process indicated by the "RAS" method we take into account the economic relationships inherent in the I-O coefficients.

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LA DINAMICA DELLE VARIAZIONI INPUT-OUTPUT

Questo articolo cerca di ottenere un nuovo metodo di revisione delle tabelle input-output prendendo in considerazione la struttura delle informazioni contenuta nell'ambiente macroeconomico. Il « filtro Bayesiano ottimale » è stato ottenuto considerando una struttura dinamica e si è derivata una equazione di aggiornamento. Questo metodo di aggiornamento è stato applicato alla matrice « input-output » di un sistema dinamico di Leontief. La struttura probabilistica desunta da variabili macroeconomiche come i vettori di consumi e produzione è stata usata per aggiornare la struttura probabilistica delle matrice « input-output ».

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THE FREE-RIDER PROBLEM: A PEDAGOGICAL NOTE
(Using Indifference Curve Technology)

by
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Introduction

The so-called "free-rider" problem is well known to students of public finance, public choice, labor markets, and other fields in economics. Indeed, this topic has recently been discussed by Asch and Gigliotti (1991). By not revealing preferences for "public goods" or other goods for which the "exclusion principle" does not apply, individuals can avoid paying their "tax-share" for the goods while still enjoying the consumption thereof.

The purpose of this brief note is to present a pedagogically useful new (albeit still rudimentary) way of teaching the free-rider concept. This simple approach applies the familiar tools of indifference curve analysis to the topic at hand. Since so many students of economics are well versed in indifference curves, this approach is suggested as a useful supplement to the usual textbook treatment, which generally adopts supply-demand analysis.

The Basic Model

We begin by observing that the economic environment in which the individual lives can be referred to as an environment of either "small numbers" or an environment of "large numbers".

We begin with the small numbers case. Let "n" represent the number of economic agents in a given environment. In the simplest case, $n = 2$. We now address this case. We let the two individuals be denoted as "A" and "B".

Individual A seeks to maximize his utility

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(1) $$U_a = f(x_a, y_a)$$

where U_a represents A's utility level, x_a is the quantity consumed of pure private good x by A, and y_a is the quantity consumed by A of public good y, where good y possesses the trait that the exclusion principle does not apply.

Individual A seeks to maximize his utility subject to the budget constraint

(2) $$DI_a = P_{y_a} y_a + P_x x_a$$

where DI_a is A's disposable income, P_x is the unit price of x, and P_{y_a} is the unit price of y paid by individual A.

Individual B faces a similar circumstance, i.e., seeks to maximize

(3) $$U_b = g(x_b, y_b)$$

subject to

(4) $$DI_b = P_{y_b} + P_x x_b$$

Given that the exclusion principle does not apply to commodity y, it follows that

(5) $$y_a = y_b = y$$

where y is either > 0 or $= 0$. For simplicity, we now assume that the *total* unit price of public good y is constant, i.e., that the supply curve of the public good is horizontal (perfectly price elastic) at the level P_y. This is a common simplifying assumption found in a variety of textbooks (Browning and Browning, 1983, p. 30; Herber, 1979, p. 55; Hyman, 1987, p. 124). Nevertheless, it should be noted that the analysis presented below can be extended to the case of a positively sloped supply curve of the public good quite easily.

The Free Rider

If one party (say, individual A) does not reveal his preference for the public good, whereas the other party (individual B, in this case) does, then $P_{y_b} > 0$ while $P_{y_a} = 0$. In addition, in this situation, the amount of y consumed by both A and B is determined solely by B. Finally, individual A's consumption of y is equal to B's (see equation (5)), despite the fact that he does not expressly pay for y.

Under these circumstances, A's budget constraint becomes

(6) $$DI_a = P_{y_a} y_a + P_x x_a = 0 y_a + P_x x_a = P_x\, x_a$$

Hence, in the traditional two-dimensional indifference curve paradigm, A's budget constraint (see panel (a) of Figure 1) then becomes perfectly vertical,

(7) $$dy/dx = - P_x/P_{y_a} = - P_x/0 = \text{undefined}$$

whereas B's budget constraint is negatively sloped (see panel (b) of Figure 1),

(8) $$dy/dx = - P_x/P_{y_b} < 0$$

Naturally, if both individuals reveal their true preferences for y, they both face negatively sloped budget lines.

The key to the analysis is determining whether an individual (A in this case) will in fact decide to be a "free-rider", i.e., not to reveal preferences for y and to thereby face an effective $P_{y_a} = 0$ while consuming those positive amounts of y determined solely by B's revealed preferences for y. It is demonstrated below that the prospect of consuming y "for free"

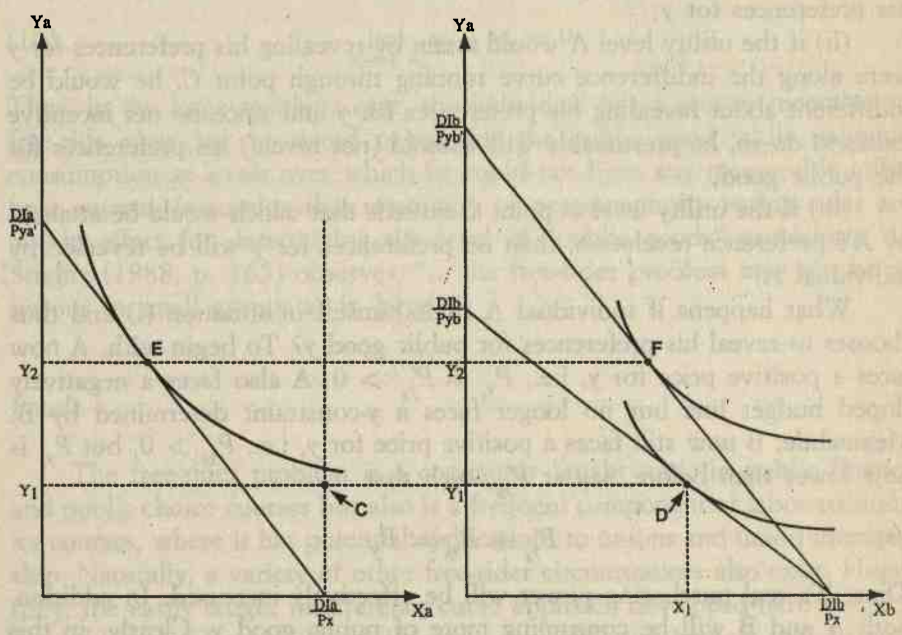

FIGURE 1.

does not necessarily imply that the free-rider option will be chosen in the small numbers case.

Refer now to Figure 1, where individuals A and B are represented in panels (a) and (b), respectively. Let us assume that B reveals his true preferences for y whereas A decides to be a free-rider. B achieves his constrained maximum utility at the tangency between an indifference curve and his budget line at point D, where he is consuming x_1 units of x and y_1 units of y. Simultaneously, A achieves his constrained maximum utility at point C along his vertical budget line, where point C corresponds to precisely y_1 units of good y. The amount y_1 in effect becomes a second constraint with respect to which individual A seeks to maximize utility if he free-rides and thusly has the amount of y he can (does) consume being strictly determined (constrained) by B's revealed preferences for y.

Will individual A in fact decide to free-ride?

At first glance, there appear to be three categories of possible scenarios:

(i) if the utility level A would attain by revealing his preferences for y, thereby having to pay a positive price for y (rather than a zero price) and also thereby facing a negatively sloped budget line but not a y-constraint, exceeds his utility level at point C, then he presumably will choose to reveal his preferences for y;

(ii) if the utility level A would attain by revealing his preferences for y were along the indifference curve running through point C, he would be indifferent about revealing his preferences for y and since no net incentive exists to do so, he presumably will withold (not reveal) his preferences for the public good;

(iii) if the utility level at point C exceeds that which would be attained by A's preference revelation, then no preferences for y will be revealed by individual A.

What happens if individual A finds himself in situation (i) and thus chooses to reveal his preferences for public good y? To begin with, A now faces a positive price for y, i.e., $P_{y_a} = P'_{y_a} > 0$. A also faces a negatively sloped budget line but no longer faces a y-constraint determined by B. Meanwhile, B now still faces a positive price for y, i.e., $P_{y_b} > 0$, but P_{y_b} is now lower than before, say at P'_{y_b}, such that

$$P'_{y_a} + P'_{y_b} = P_y \tag{9}$$

Thus, B's real purchasing power will be effectively increased. In addition, both A and B will be consuming more of public good y. Clearly, in this situation, both parties can in theory attain a higher level of utility.

To demonstrate the kind of solution that potentially can be reached under scenario (i), refer again to Figure 1. In this scenario, individual A now faces a negatively sloped budget line (that still runs through the original coordinates $(DI_a/P_x, 0)$) and reaches a new consumer equilibrium at a point such as E in panel (a). B now faces a new ("higher", rotated) budget line, that runs through the original coordinates $(DI_b/P_x, 0)$ and reaches a new consumer equilibrium at a point such as F in panel (b).

Clearly, both parties consume more of public good y than originally. In A's case, his consumption as a free-rider was y_1, but now it is y_2. Given the nature of good y, B likewise has experienced a rise in public good consumption in the amount of $y_2 - y_1$. As shown in the panels, both parties can achieve higher utility levels in this type of circumstance.

Clearly, in the very small numbers case, where the individual can measurably influence the outcome, i.e., can measurably influence the amount of the public good provided, the individual *may* have an incentive to reveal preferences even though doing so he increases his "tax burden" or outlay level. In the example provided above, A's consumption increase in terms of the public good y rose by the amount $y_2 - y_1$. But as the number of parties in the environment rises, the potential influence of the individual on the output level of the public good declines, such that

$$\lim_{n \to +\infty} y_2 - y_1 = 0 \qquad (10)$$

Thus, in the large numbers case, the individual has a greater incentive to free-ride since he can avoid paying for the public good while enjoying consumption at levels over which he could not have any measurable influence anyway (assuming that unanimity or near-unanimity voting rules are not in effect for determining the level of "public goods" provision). As Stiglitz (1988, p. 165) observes, "... the free-rider problem may not be as serious in small groups as in large...".

Finale

The free-rider problem is a commonly taught topic in public finance and public choice courses but also is a frequent component of labor economics courses, where it has potential applications to unions and union membership. Naturally, a variety of other free-rider circumstances also exist. Hopefully, the easily taught indifference curve approach developed here to elucidate the free-rider problem will prove a useful pedagogical tool to supplement the traditional supply-demand paradigm. Moreover, this approach can be easily modified to allow for more complex circumstances. For example, if there are direct transactions costs to revealing preferences for "public goods", as might be the case for voting, then such costs could be included in the *total* price of the public good.

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IL PROBLEMA DEL FREE RIDER: NOTA PEDAGOGICA

Questo articolo usa l'analisi delle curve di indifferenza per mostrare la dinamica del problema del free rider. L'analisi mostra le circostanze in cui, nel caso di piccoli numeri, è razionale per un individuo indicare o no le sue vere preferenze per i beni pubblici. Questa analisi sostituisce l'esposizione eccessivamente semplicistica del problema del free rider basata sulla domanda e l'offerta.

RECENSIONI

JENKIS Helmut: *Sozialutopien - barbarische Glucksverheissungen? Zur Geistesgeschichte der Idee von der vollkommenen Gesellschaft.* 1992, Berlin, Duncker & Humblot, pp. XX-535.

Con la stesura di codesto volume, che ha per sottotitolo « Per una storia intellettuale dell'idea della società perfetta » e che si snoda per oltre cinquecento pagine, l'Autore ha seguito un iter comune a molti economisti. Dopo una vita di studio e di ricerca pura o applicata nell'orto della « dismal science », che ricorda a tutti la limitatezza dei mezzi rispetto ai bisogni, quando i capelli ingrigiscono o sbiancano e gli occhi abbisognano degli occhiali, l'economista volge la propria attenzione e riflessione alla filosofia sociale e politica. A volte propone le proprie soluzioni o visioni dell'uomo, della società o dello stato, perché anche l'economista può (e deve) « avere un sogno ». A volte, come nel caso in esame, ripercorre una « Geistesgeschichte », una storia di idee che, quando hanno avuto pratica applicazione, hanno portato alla barbarie, alla negazione dell'uomo. Non avanziamo un'ipotesi ma esponiamo molto più di un'opinione dicendo che due sono le ragioni che hanno indotto l'Autore a compiere questa autentica fatica. In primo luogo l'essere un economista vero, conscio del fatto che « nessun pasto è gratis » ma anche che, come dice Einaudi, ci sono uomini che inventano, faticano, producono non per meschino egoismo materiale, ma per realizzare se stessi giovando nel contempo agli altri. In secondo luogo la sua storia personale. Da ragazzo ha vissuto il nazionalsocialismo e la triste avventura del profugo davanti all'Armata Rossa avanzante, lasciando la baltica natìa Memel, piccola isola germanica nel mare slavo. Conoscere e interpretare a fondo le basi teoriche della follia che ha comportato la distruzione e lo smembramento per quasi quarantanni del suo paese, e di quel regime che ha promesso « l'uomo nuovo », ecco il secondo motivo di un libro, dedicato ai giovani, e del quale si consiglia la traduzione (e non solo in italiano) perché sarebbe un validissimo strumento di lavoro per gli studenti (e i docenti) di parecchie Facoltà. Si tratta infatti di un vero manuale, in cui i pensatori e le loro teorie e realizzazioni sono inquadrati nel rispettivo tempo e spazio con chiarezza e rigore, in un tedesco « nordico » che va particolarmente apprezzato ai giorni nostri, in cui anche la lingua di Goethe è crescentemente farcita di inglesismi e nelle sue espressioni correnti e più superficiali, anche di italianismi stucchevoli.

Dopo un primo capitolo dedicato a « Concetti e forme delle Utopie », nei successivi capitoli, l'Autore effettua la rassegna delle utopie politiche (Staatsutopien) e sociali. Per prime – nel II capitolo – vengono esaminate le utopie « classiche », da Diodoro Siculo a Platone, a Tomaso Moro, il padre delle utopie politiche, a Tommaso Campanella, all'utopista protestante Johann V. Andreae, sicuramente meno noto ma non meno interessante e a F. Bacone. Nel III capitolo vengono trattate le utopie ispirate a un radicalismo rivoluzionario d'origine religiosa, che non sono rimaste costruzioni intellettuali come quelle classiche ma hanno cercato di realizzarsi anche attraverso rivoluzioni e guerre, come le utopie di T. Münzer e degli Anabattisti e lo stato dei Gesuiti in Paraguay, finito anch'esso nel sangue. Se il '700, il secolo dei lumi, non produce utopie (i migliori ingegni erano, in complesso, occupati in cose più importanti!), è nel 1800, nel mutato contesto economico e sociale che abbiamo il rifiorire delle utopie a maggiore connotazione economico-sociale, quali quelle di Owen, Fourier, e Cabet, i cui tentativi di realizzazione si sono estinti senza sangue e rovine ma per asfissia. Se il contenuto dei capitoli II, III e IV vanno valutati e apprezzati come una precisa e acuta ricostruzione della storia delle Utopie, i capitoli V e VI dedicati rispettivamente al marxismo e al mito della razza ariana costituiscono i « pezzi forti » del volume. Le utopie della società senza classi e della razza pura, hanno potuto avere realizzazione in due grandi paesi, e sono stati proprio le modalità e i risultati di essa, che impongono di definire le due suddette utopie come « promesse di felicità barbariche » (barbarische Glucksverheissungen). La sconfitta sul campo del marxismo realizzato deriva proprio dal carattere utopico, sia del marxismo, sia del socialismo pensato e scientifico, dall'impossibilità cioè di rivoltare la natura umana. Per realizzarsi ha dovuto ricorrere alle repressioni di massa, alle purghe, al terrore, all'apertura di manicomi per i dissidenti. Il tragico spegnersi delle illusioni di benessere materiale, di libera esplicazione della personalità di uomini nuovi, dimostrano, secondo l'Autore, che di utopia barbarica e non di scienza si è trattato.

Se il contenuto del capitolo VI è da apprezzare per la logica stringente con la quale il discorso si snoda fino alla conclusione, quello del capitolo VII dedicato alla « barbarica utopia razzista del nazionalsocialismo » lo è ancora di più, perché tratta con rigore e ricchezza di particolari due aspetti sicuramente meno o più superficialmente noti. Il primo è la « fontana di vita » ossia la procreazione di bimbi ariani mediante giovani donne che mettono il proprio grembo a disposizione della razza pura e della patria per essere fecondate dai « guerrieri » del III Reich germanico. Sono pagine che ispirano profonda tristezza, che è comunque meno dell'orrore suscitato da quelle dedicate al secondo aspetto, quello dell'eutanasia da praticare sulle persone malate e tarate in modo irrecuperabile.

In chiusura vale la pena di riportare le parole del grande Blaise Pascal, che l'Autore mette in epigrafe al libro: « L'uomo è né un angelo né una fiera e la sua sfortuna è che egli diventa tanto più feroce quanto più vuole essere un angelo ». Siccome questo è purtroppo vero, l'economista può esclamare: « viva i massimi e i minimi vincolati », che possono dare sicuramente non una irreale felicità ma la serena coscienza del dovere compiuto, ch'è l'unico paradiso in terra che l'uomo può permettersi.

Davide Cantarelli

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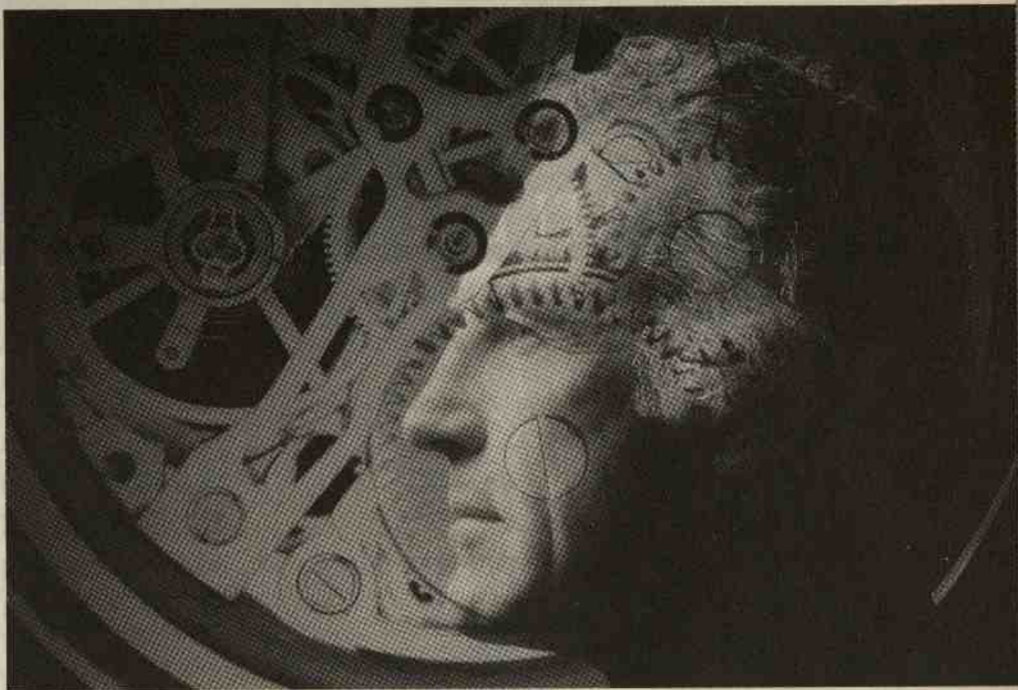

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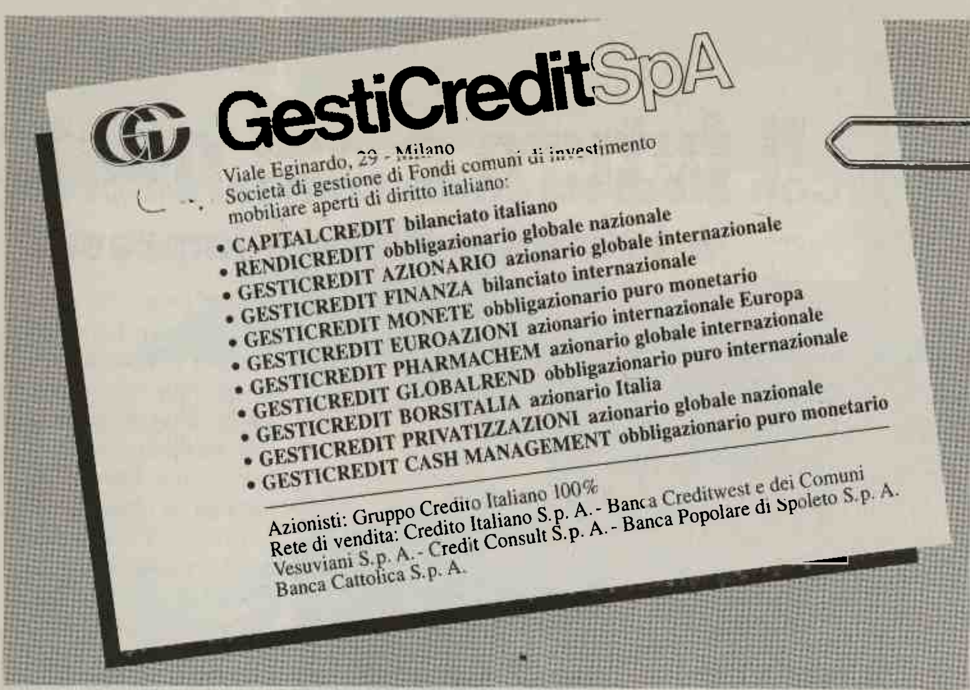

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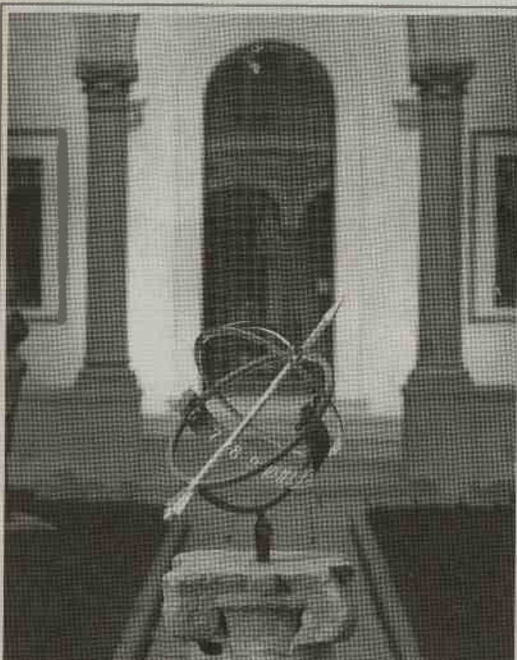
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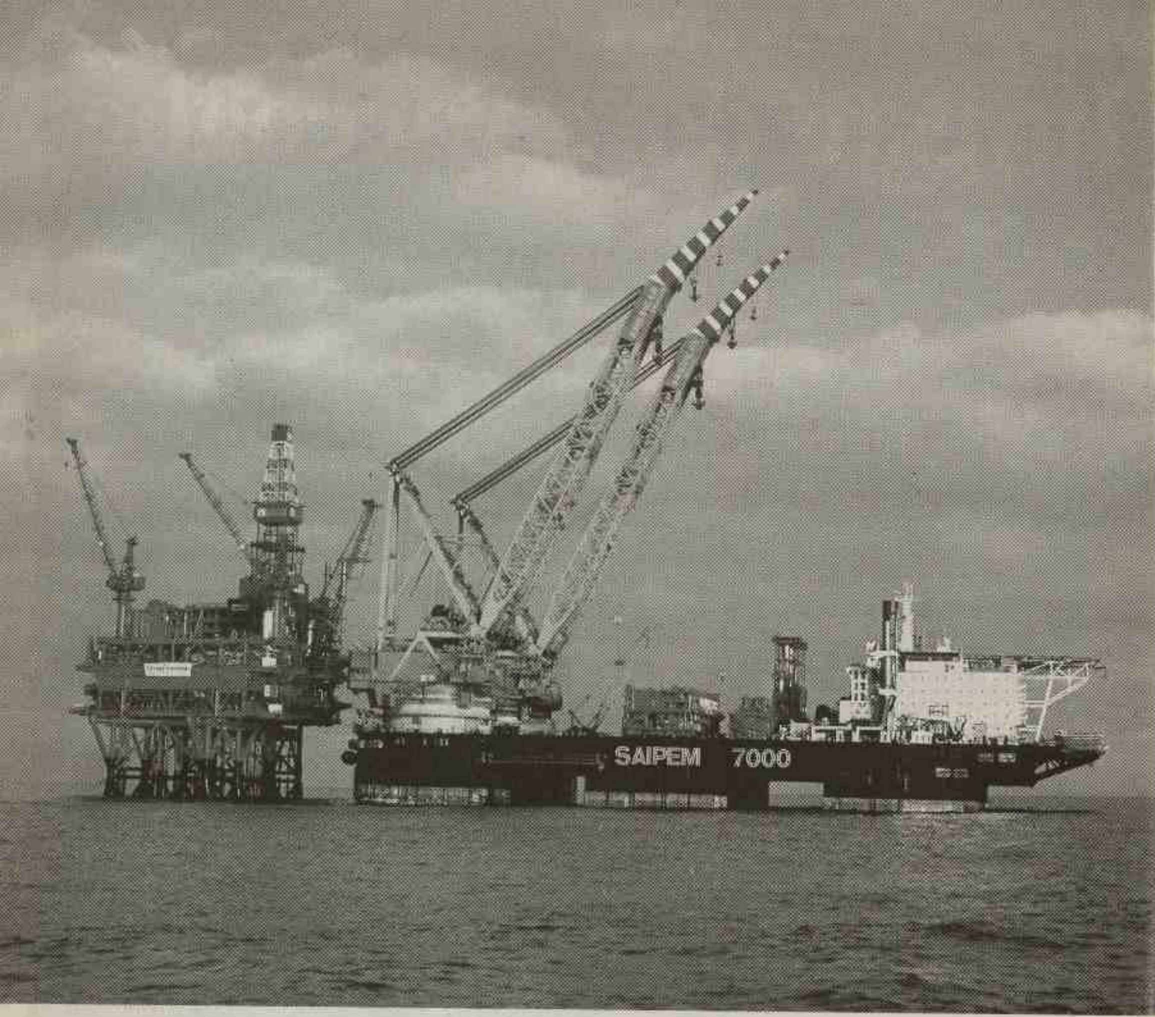

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